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SOLUTION
MAGICAL MATHS
8

1. Rational Numbers

Exercise 1.1

1. Find the value :

(a) $\left|\frac{-3}{17}\right| = \frac{3}{17}$

(b) $\left|\frac{-14}{13}\right| - \left|\frac{-3}{20}\right|$

$= \frac{14}{13} - \frac{3}{20}$

$= \frac{280 - 39}{260}$

$= \frac{241}{260}$

(c) $\left|\frac{7}{12} + \frac{-9}{12}\right|$

$= \left|\frac{7}{12} - \frac{9}{12}\right|$

$= \left|\frac{7 - 9}{12}\right|$

$= \left|\frac{-2}{12}\right|$

$= \frac{2}{12} = \frac{1}{6}$

2. Fill in the blanks with <, > or =.

(a) $\frac{7}{-13}$ $\boxed{<}$ $\frac{8}{15}$

$\Rightarrow$ $\frac{-7}{13} \times \frac{8}{15}$

$\Rightarrow$ $-105 < 104$

(b) $\frac{5}{8}$ $\boxed{>}$ $\frac{-9}{14}$

$\Rightarrow$ $\frac{5}{8} \times \frac{-9}{14}$

$\Rightarrow$ $70 > -72$

(c) $\frac{-11}{16}$ $\boxed{>}$ $\frac{13}{-16}$

$\Rightarrow$ $\frac{-11}{16} \times \frac{-13}{16}$

$\Rightarrow$ $-176 > -208$

3. Rewrite the following in ascending order :

(a) $\frac{2}{3}, \frac{4}{9}, \frac{-4}{15}, \frac{7}{9}$

LCM of 3, 9, 15 and 9 = 45

$\Rightarrow \quad \frac{2}{3} = \frac{2\times 15}{3\times 15} = \frac{30}{45}, \qquad \frac{4}{9} = \frac{4\times 5}{9\times 5} = \frac{20}{45}$

$\frac{-4}{15} = \frac{-4\times 3}{15\times 3} = \frac{-12}{45}$ and $\frac{7}{9} = \frac{7\times 5}{9\times 5} = \frac{35}{45}$

Ascending order is :

$\frac{-12}{45} < \frac{20}{45} < \frac{30}{45} < \frac{35}{45}$

$\therefore \quad \frac{-4}{15} < \frac{4}{9} < \frac{2}{3} < \frac{7}{9}$

(b) $\frac{9}{25}, \frac{2}{5}, \frac{14}{-75}, \frac{-19}{10}$

$= \frac{9}{25}, \frac{2}{5}, \frac{-14}{75}, \frac{-19}{10}$

$\therefore$ LCM of 25, 5, 75 and 10 = 150

$\frac{9}{25} = \frac{9\times 6}{25\times 6} = \frac{54}{150}, \qquad \frac{2}{5} = \frac{2\times 30}{5\times 30} = \frac{60}{150}$

$\frac{-14}{75} = \frac{-14\times 2}{75\times 2} = \frac{-28}{150}$ and $\frac{-19}{10} = \frac{-19\times 15}{10\times 15} = \frac{-285}{150}$

Ascending order is :

$\frac{-285}{150} < \frac{-28}{150} < \frac{54}{150} < \frac{60}{150}$

$\therefore \quad \frac{-19}{10} < \frac{-14}{75} < \frac{9}{25} < \frac{2}{5}$

4. Rewrite the following in descending order :

(a) $\frac{19}{24}, \frac{5}{6}, \frac{5}{30}, \frac{-17}{18}$

$\therefore$ LCM of 24, 6, 30 and 18 = 360

$\frac{19}{24} = \frac{19\times 15}{24\times 15} = \frac{285}{360}, \quad \frac{5}{6} = \frac{5\times 60}{6\times 60} = \frac{300}{360}$

$\frac{5}{30} = \frac{5\times 12}{30\times 12} = \frac{60}{360}$ and $\frac{-17}{18} = \frac{-17\times 20}{18\times 20} = \frac{-340}{360}$

Descending order :

$\frac{300}{360} > \frac{285}{360} > \frac{60}{360} > \frac{-340}{360}$

$\therefore \quad \frac{5}{6} > \frac{19}{24} > \frac{5}{30} > \frac{-17}{18}$

(b) $\frac{7}{8}, \frac{11}{12}, \frac{15}{-16}, \frac{1}{4} \quad \Rightarrow \quad \frac{7}{8}, \frac{11}{12}, \frac{-15}{16}, \frac{1}{4}$

$\therefore$ LCM of 8, 12, 16 and 4 = 48

$$\frac{7}{8} = \frac{7 \times 6}{8 \times 6} = \frac{42}{48}, \quad \frac{11}{12} = \frac{11 \times 4}{12 \times 4} = \frac{44}{48}$$

$$\frac{-15}{16} = \frac{-15 \times 3}{16 \times 3} = \frac{-45}{48} \quad \text{and} \quad \frac{1}{4} = \frac{1 \times 12}{4 \times 12} = \frac{12}{48}$$

Descending order

$$\frac{44}{48} > \frac{42}{48} > \frac{12}{48} > \frac{-45}{48}$$

$$\frac{11}{12} > \frac{7}{8} > \frac{1}{4} > \frac{-15}{16}$$

5. Simplfy the following :

(a) $\frac{-8}{15} + \frac{-7}{15}$

$$= \frac{-8}{15} - \frac{7}{15}$$

$$= \frac{-8 - 7}{15}$$

$$= \frac{-15}{15}$$

$$= -\mathbf{1}$$

(b) $\frac{1}{6} + \frac{-2}{3} - \frac{1}{3}$

$$= \frac{1}{6} - \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1 - 4 - 2}{6}$$

$$= \frac{1 - 6}{6}$$

$$= \mathbf{\frac{-5}{6}}$$

(c) $\frac{5}{12} - \frac{7}{36}$

$$= \frac{15 - 7}{36}$$

$$= \frac{8}{36}$$

$$= \mathbf{\frac{2}{9}}$$

(d) $\frac{21}{25} - \frac{16}{5} + \frac{7}{25}$

$$= \frac{21 - 80 + 7}{25}$$

$$= \frac{28 - 80}{25}$$

$$= \mathbf{\frac{52}{25}}$$

(e) $-5 + \frac{-9}{16} + \frac{-5}{8}$

$$= \frac{-5}{1} - \frac{9}{16} - \frac{5}{8}$$

$$= \frac{-80 - 9 - 10}{16}$$

$$= \mathbf{\frac{-99}{16}}$$

6. Evaluate the following :

(a) $\frac{8}{-9} \times \frac{15}{-7}$

(b) $\frac{-11}{9} \times \frac{-81}{88}$

$= \frac{8}{-3} \times \frac{5}{-7}$

$= \mathbf{\frac{40}{21}}$

$= \frac{-1}{1} \times \frac{-9}{8}$

$= \mathbf{\frac{9}{8}}$

(c) $\frac{5}{7} \times \frac{-14}{25} \times \frac{15}{8}$

$= \frac{1}{1} \times \frac{-2}{5} \times \frac{15}{8}$

$= \mathbf{\frac{-3}{4}}$

(d) $\frac{-15}{19} \div \frac{30}{27}$

$= \frac{-15}{19} \times \frac{27}{30}$

$= \mathbf{\frac{-27}{38}}$

(e) $\frac{-25}{7} \div \frac{15}{14}$

$= \frac{-25}{7} \times \frac{14}{15} = \frac{-5}{1} \times \frac{2}{3}$

$= \mathbf{\frac{-10}{3}}$

Exercise 1.2

1. Write the additive inverse of the following :

(a) $\frac{-6}{13}$ Additive inverse of $\frac{-6}{13} = \frac{6}{13}$

$\because \quad \frac{-6}{13} + \frac{6}{13} = 0$

(b) -8 additive inverse of $-8 = 8$

$\because \quad -8 + 8 = 0$

(c) $\frac{4}{11}$ additive inverse of $\frac{4}{11} = \frac{-4}{11}$

$\because \quad \frac{4}{11} + \frac{-4}{11} = 0$

2. Write the multiplicative inverse of the following :

(a) $\frac{-16}{19}$ Multiplicative inverse of $\frac{-16}{19} = \frac{19}{-16}$

$\because \quad \frac{-16}{19} \times \frac{19}{-16} = 1$

(b) -12 multiplicative inverse of $-12 = \frac{1}{-12}$

$\because \quad -12 \times \frac{1}{-12} = 1$

(c) $\frac{5}{9}$ multiplicative inverse of $\frac{5}{9} = \frac{9}{5}$

$\because \quad \frac{5}{9} \times \frac{9}{5} = 1$

3. Verify the property $a \times (b \times c) = (a \times b) \times c$ by taking :

(a) $a = \frac{-7}{3}, b = \frac{5}{-4}, c = \frac{9}{8}$

$$a \times (b \times c) = (a \times b) \times c$$

$$\frac{-7}{3} \times \left(\frac{5}{-4} \times \frac{9}{8}\right) = \left(\frac{-7}{3} \times \frac{5}{-4}\right) \times \frac{9}{8}$$

$$\frac{-7}{3} \times \left(\frac{45}{-32}\right) = \left(\frac{-35}{-12}\right) \times \frac{9}{8}$$

$$\frac{-7}{3} \times \frac{45}{-32} = \frac{35}{12} \times \frac{9}{8}$$

$$\frac{-315}{-96} = \frac{315}{96}$$

$$\frac{315}{96} = \frac{315}{96}$$

$\therefore$ LHS = RHS

(b) $a = \frac{2}{7}, b = \frac{-7}{3}, c = \frac{4}{9}$

$$a \times (b \times c) = (a \times b) \times c$$

$$\frac{2}{7} \times \left(\frac{-7}{3} \times \frac{4}{9}\right) = \left(\frac{2}{7} \times \frac{-7}{3}\right) \times \frac{4}{9}$$

$$\frac{2}{7} \times \left(\frac{-28}{27}\right) = \frac{-14}{21} \times \frac{4}{9}$$

$$\frac{-8}{27} = \frac{-8}{27}$$

$\therefore$ LHS = RHS

4. Evaluate the following using distributive property :

(a) $\frac{7}{9} \times \left(\frac{-1}{2}\right) - \frac{7}{9} \times \left(\frac{-3}{2}\right)$

$= \frac{7}{9} \times \left[\frac{-1}{2} - \left(\frac{-3}{2}\right)\right]$

(distributive property)

$= \frac{7}{9} \times \left(\frac{-1}{2} + \frac{3}{2}\right)$

$= \frac{7}{9} \times \left(\frac{-1+3}{2}\right)$

$= \frac{7}{9} \times \frac{2}{2}$

$= \frac{7}{9}$

(b) $\left[\left(\frac{-2}{7}\right) \times \left(\frac{-4}{9}\right)\right] + \left[\left(\frac{-3}{5}\right) \times \left(\frac{-4}{9}\right)\right]$

$= \left(\frac{-4}{9}\right) \times \left[\left(\frac{-2}{7}\right) + \left(\frac{-3}{5}\right)\right]$

(distributive property)

$= \left(\frac{-4}{9}\right) \times \left[\frac{-2}{7} - \frac{3}{5}\right]$

$= \left(\frac{-4}{9}\right) \times \left[\frac{-10-21}{35}\right]$

$= \frac{-4}{9} \times \frac{-31}{35}$

$= \frac{124}{315}$

(c) $\left(\frac{3}{8} \times \frac{8}{10}\right) - \left[\left(\frac{-7}{9}\right) \times \frac{3}{8}\right] + \left[\frac{3}{8} \times \left(\frac{-4}{5}\right)\right]$

$= \frac{3}{8} \times \left[\frac{8}{10} - \left(\frac{-7}{9}\right) + \left(\frac{-4}{5}\right)\right]$

$= \frac{3}{8} \times \left[\frac{8}{10} + \frac{7}{9} - \frac{4}{5}\right]$

$= \frac{3}{8} \times \left[\frac{72+70-72}{90}\right]$

$= \frac{3}{8} \times \frac{70}{90} = \frac{7}{24}$

(d) $\frac{-7}{3} \times \left(\frac{12}{5} - \frac{4}{21}\right)$

$= \frac{-7}{3} \times \left(\frac{252-20}{105}\right)$

$= \frac{-7}{3} \times \frac{232}{105}$

$= \frac{-232}{45}$

5. Simplify the following :

(a) $\frac{5}{12}\times\frac{1}{40}+\frac{7}{9}\times\frac{3}{5}+\frac{7}{9}\times\left(\frac{-5}{8}\right)$

$=\frac{5}{12}\times\frac{1}{40}+\frac{7}{9}\times\frac{3}{5}+\frac{7}{9}\times\frac{(-5)}{8}$

$=\frac{1}{96}+\frac{7}{15}-\frac{35}{72}$

$=\frac{15+672-700}{1440}$

$=\frac{687-700}{1440}$

$=\frac{-13}{1440}$

(b) $\frac{11}{12}\times\frac{-7}{8}\times\frac{-8}{15}\times\frac{9}{-11}$

$=\frac{11}{12}\times\frac{-7}{8}\times\frac{-8}{15}\times\frac{9}{-11}$

$=\frac{1}{12}\times\frac{-7}{1}\times\frac{-1}{5}\times\frac{3}{-1}$

$=\frac{-7\times 3}{12\times 5}$

$=\frac{-7}{20}$

(c) $\left[\left(\frac{-8}{5}\right)\times\frac{3}{4}\right]+\left[\frac{7}{8}\times\left(\frac{-16}{25}\right)\right]$

$=\left(\frac{-8}{5}\times\frac{3}{4}\right)+\left(\frac{7}{8}\times\frac{-16}{25}\right)$

$=\frac{-6}{5}+\left(\frac{-14}{25}\right)$

$=\frac{-30-14}{25}$

$=\frac{-44}{25}$

(d) $\frac{36}{49}\div\frac{27}{-42}\times\frac{26}{-45}\div\frac{65}{-81}$

$=\frac{36}{49}\times\frac{-42}{27}\times\frac{26}{-45}\times\frac{-81}{65}$

$=\frac{4}{7}\times\frac{-6}{3}\times\frac{2}{-5}\times\frac{-9}{5}$

$=\frac{144}{-175}$

$=-\frac{144}{175}$

(e) $\left(\frac{2}{9}\times\frac{3}{6}\right)+\left(\frac{3}{4}\div\frac{12}{5}\right)-\left[\frac{1}{4}-\left(\frac{1}{4}\times\frac{2}{3}\right)\right]$

$=\left(\frac{2}{9}\times\frac{3}{6}\right)+\left(\frac{3}{4}\times\frac{5}{12}\right)-\left(\frac{1}{4}-\frac{1}{6}\right)$

$=\frac{1}{9}+\frac{5}{16}-\left(\frac{3-2}{12}\right)$

$=\frac{1}{9}+\frac{5}{16}-\frac{1}{12}$

$=\frac{32+90-24}{288}$

$=\frac{98}{288}=\frac{49}{144}$

6. Sum of $\frac{2}{9}$ and $\frac{4}{7} = \frac{2}{9} + \frac{4}{7}$

$$= \frac{14+36}{63}$$

$$= \frac{50}{63}$$

Difference of $\frac{2}{9}$ and $\frac{4}{7} = \frac{2}{9} - \frac{4}{7}$

$$= \frac{14-36}{63}$$

$$= \frac{-22}{63}$$

Now, $\frac{50}{63} \div \left(\frac{-22}{63}\right) = \frac{50}{63} \times \frac{63}{-22}$

$$= \frac{50}{-22}$$

$$= \frac{-25}{11}$$

7. Let the required number be x

$\therefore \quad x \times \frac{-15}{16} = \frac{5}{6}$

$$x = \frac{5}{6} \div \frac{-15}{16}$$

$$= \frac{5}{6} \times \frac{16}{-15}$$

$$= \frac{8}{-9}$$

$\therefore$ Other number $= \frac{-8}{9}$

8. Total quantity of oil $= 84\frac{4}{5}$ litres

$$= \frac{424}{5} \text{ litres}$$

Quantity of oil filled in container $= 11\frac{2}{3}$ litres

$$= \frac{35}{3} \text{ litres}$$

Quantity of oil which can be hold more by the container $= \frac{424}{5} - \frac{35}{3}$

$$= \frac{1272-175}{15} \text{ litres}$$

$$= \frac{1097}{15} \text{ litres}$$

$$= 73\frac{2}{15} \text{ litres}$$

9. Verify the following and name the property used in each case :

(a) $\frac{11}{14} \times \frac{-2}{5} = \left(\frac{-2}{5}\right) \times \frac{11}{14}$

$\frac{11}{14} \times \frac{-2}{5} = \frac{-2}{5} \times \frac{11}{14}$ (Commutative property for multiplication)

$\frac{-11}{35} = \frac{-11}{35}$

$\therefore$ LHS = RHS

(b) $\frac{7}{9} \times 1 = \frac{7}{9}$

$\frac{7}{9} = \frac{7}{9}$ (Multiplicative identity)

$\therefore$ LHS = RHS

(c) $\left(\frac{-9}{11}\right) + \frac{2}{5} = \frac{2}{5} + \left(\frac{-9}{11}\right)$

$\frac{-9}{11} + \frac{2}{5} = \frac{2}{5} - \frac{9}{11}$ (Commutative property for addition)

$\frac{-45+22}{55} = \frac{22-45}{55}$

$\frac{-23}{55} = \frac{-23}{55}$

$\therefore$ LHS = RHS

(d) $\left(\frac{3}{7} \times \frac{4}{9}\right) \times \left(\frac{-7}{12}\right) = \frac{3}{7} \times \left[\frac{4}{9} \times \left(\frac{-7}{12}\right)\right]$

$\left(\frac{3}{7} \times \frac{4}{9}\right) \times \left(\frac{-7}{12}\right) = \frac{3}{7} \times \left[\frac{4}{9} \times \frac{(-7)}{12}\right]$

$\frac{4}{21} \times \frac{(-7)}{12} = \frac{3}{7} \times \frac{(-7)}{27}$ (Associative property for multiplication)

$\frac{-1}{9} = \frac{-1}{9}$

$\therefore$ RHS = RHS

(e) $\frac{-5}{21} + 0 = \frac{-5}{21}$

$\frac{-5}{21} = \frac{-5}{21}$ (Additive identity)

$\therefore$ LHS = RHS

(f) $\frac{-5}{21} \times \frac{21}{-5} = 1$

$\frac{-5}{21} \times \frac{21}{-5} = 1$ (Multiplicative identity)

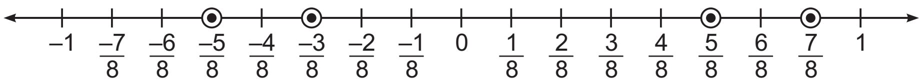
$1 = 1$

$\therefore$ LHS = RHS

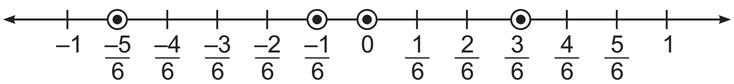
Exercise 1.3

1. Represent the following rational numbers on the number line :

(a) $\frac{-5}{8}, \frac{-3}{8}, \frac{5}{8}, \frac{7}{8}$



(b) $\frac{-1}{6}, \frac{-5}{6}, \frac{0}{6}, \frac{3}{6}$



2. Which of the following rational numbers are to the right of zero and which of them are to the left of zero ?

(a) $\frac{-3}{5}$ $\qquad$ $\frac{-3}{5}$ is a negative rational number, so it is to the left of zero.

(b) $\frac{1}{10}$ $\qquad$ $\frac{1}{10}$ is a positive rational number so it is to the right of zero.

3. Five rational numbers between 0 and 1.

$\therefore$ $\qquad$ $0 = \frac{0}{10}$

and $\qquad$ $1 = \frac{10}{10}$

So, five rational numbers between $\frac{0}{10}$ and $\frac{10}{10}$ are :

$$\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}$$

4. Find any two rational numbers between :

(a) $\frac{-3}{4}$ and $\frac{-4}{5}$

$$\frac{-3}{4} = \frac{-3 \times 15}{4 \times 15} = \frac{-45}{60}$$

and $\qquad$ $\frac{-4}{5} = \frac{-4 \times 12}{5 \times 12} = \frac{-48}{60}$

So, two rational numbers between $\frac{-45}{60}$ and $\frac{-48}{60}$ are :

$$\frac{-46}{60}, \frac{-47}{60}$$

(b) $\frac{2}{7}$ and 2

$$\frac{2}{7} = \frac{2 \times 2}{7 \times 2} = \frac{4}{14}$$

and $\qquad$ $2 = \frac{2 \times 14}{1 \times 14} = \frac{28}{14}$

So, two rational numbers between $\frac{4}{14}$ and $\frac{28}{14}$ are :

$$\frac{5}{14}, \frac{27}{14}$$

(c) $\frac{1}{3}$ and $\frac{1}{2}$

$$\frac{1}{3} = \frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

and $$\frac{1}{2} = \frac{1 \times 9}{2 \times 9} = \frac{9}{18}$$

So, two rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are :

$$\frac{7}{18}, \frac{8}{18}$$

5. Point A represents $\frac{-13}{5}$ as a rational number.

2. Exponents & Power

Exercise 2.1

1. Evaluate :

(a) $\left(\frac{1}{4}\right)^{-3}$

$= (4)^3$

$= 4 \times 4 \times 4$

$= 64$

(b) $(-5)^{-4} = \left(\frac{-1}{5}\right)^4$

$= \left(\frac{-1}{5}\right) \times \left(\frac{-1}{5}\right) \times \left(\frac{-1}{5}\right) \times \left(\frac{-1}{5}\right)$

$= \frac{1}{625}$

(c) $(-3)^{-1} \times \left(\frac{1}{3}\right)^{-1}$

$= \left(\frac{-1}{3}\right)^1 \times (3)^1$

$= \frac{-1}{3} \times 3$

$= -1$

(d) $\left(\frac{-3}{7}\right)^2$

$= \left(\frac{-3}{7}\right) \times \left(\frac{-3}{7}\right)$

$= \frac{9}{49}$

(e) $\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$= (3)^2 + (2)^2 + (4)^2$

$= 9 + 4 + 16$

$= 29$

2. Express the following with a positive exponent :

(a) 3^{-5}

$= \left(\frac{1}{3}\right)^5$

(b) $\left(\frac{1}{5}\right)^{-7}$

$= (5)^7$

(c) $\left(\frac{-2}{3}\right)^{-2}$

$= \left(\frac{-3}{2}\right)^2$

(d) $5^{-3} \times 5^{-6}$

$= \left(\frac{1}{5}\right)^3 \times \left(\frac{1}{5}\right)^6$

3. Write the reciprocal of the following :

(a) $\left(\frac{5}{7}\right)^{-4}$

Reciprocal of $\left(\frac{5}{7}\right)^{-4}$

$= \left(\frac{5}{7}\right)^4$

(b) $\left(\frac{12}{19}\right)^{-10}$

Reciprocal of $\left(\frac{12}{19}\right)^{-10}$

$= \left(\frac{12}{19}\right)^{10}$

(c) $\left(\frac{a}{b}\right)^n$

Reciprocal of $\left(\frac{a}{b}\right)^n$

$= \left(\frac{a}{b}\right)^{-n}$

(d) $\left(\frac{-8}{13}\right)^3$

Reciprocal of $\left(\frac{-8}{13}\right)^3$

$= \left(\frac{-8}{13}\right)^{-3}$

4. Write the numbers in exponential form :

(a) 81

$81 = 3 \times 3 \times 3 \times 3 = (3)^4$

(b) $\left(\frac{-343}{64}\right)$

$\frac{(-343)}{64} = \frac{(-7) \times (-7) \times (-7)}{4 \times 4 \times 4} = \left(\frac{-7}{4}\right)^3$

(c) $\frac{64}{121}$

$\frac{64}{121} = \frac{8}{11} \times \frac{8}{11} = \left(\frac{8}{11}\right)^2$

(d) $\frac{1331}{1728}$

$\frac{1331}{1728} = \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} = \left(\frac{11}{12}\right)^3$

5. Simplify :

(a) $(4^{-1} \times 2^5)$

$= \left(\frac{1}{4} \times 32\right) = 8$

(b) $(2^{-1} - 3^{-1})^{-1}$

$= \left(\frac{1}{2} - \frac{1}{3}\right)^{-1} = \left(\frac{3-2}{6}\right)^{-1}$

$= \left(\frac{1}{6}\right)^{-1} = (6)^1 = 6$

(c) $\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$

$$= \left(\frac{8}{3}\right)^{2} \times \left(\frac{5}{4}\right)^{3}$$

$$= \frac{64}{9} \times \frac{125}{64}$$

$$= \frac{125}{9}$$

(d) $\left(\frac{1}{3}\right)^{-2} \div (9)^2$

$$= (3)^2 \times \left(\frac{1}{9}\right)^{2}$$

$$= 9 \times \frac{1}{81}$$

$$= \frac{1}{9}$$

(e) $(6^{-1} - 8^{-1}) + (2^{-1} - 3^{-1})^{-1}$

$$= \left(\frac{1}{6} - \frac{1}{8}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)^{-1}$$

$$= \left(\frac{4-3}{24}\right) + \left(\frac{3-2}{6}\right)^{-1}$$

$$= \frac{1}{24} + \left(\frac{1}{6}\right)^{-1}$$

$$= \frac{1}{24} + (6)^{1}$$

$$= \frac{1}{24} + \frac{6}{1}$$

$$= \frac{1+144}{24}$$

$$= \frac{145}{24}$$

(f) $(2^0 + 4^{-1}) \times 2^2$

$$= \left(1 + \frac{1}{4}\right) \times 4$$

$$= \left(\frac{4+1}{4}\right) \times 4$$

$$= \frac{5}{4} \times 4$$

$$= 5$$

(g) $\left[\left(\frac{6}{11}\right)^{-2} \times \left(\frac{22}{5}\right)^{-2}\right] \div \left(\frac{3}{5}\right)^{-3}$

$$= \left[\left(\frac{11}{6}\right)^{2} \times \left(\frac{5}{22}\right)^{2}\right] \div \left(\frac{5}{3}\right)^{3}$$

$$= \left(\frac{11\times 11}{6\times 6} \times \frac{5\times 5}{22\times 22}\right) \times \left(\frac{3}{5}\right)^{3}$$

$$= \frac{25}{144} \times \frac{27}{125}$$

$$= \frac{3}{80}$$

6. Expand the following numbers using exponents :

(a) 3005.26

$$= 3\times 1000 + 5\times 1 + 2\times \frac{1}{10} + \frac{6\times 1}{100}$$

$$= 3\times 10^3 + 5\times 10^0 + 2\times 10^{-1} + 6\times 10^{-2}$$

(b) $3035.789 = 3\times 1000 + 3\times 10 + 5\times 1 + 7\times \frac{1}{10} + 8\times \frac{1}{100} + 9\times \frac{1}{1000}$

$= 3\times 10^3 + 3\times 10^1 + 5\times 10^0 + 7\times 10^{-1} + 8\times 10^{-2} + 9\times 10^{-3}$

(c) $0.1234 = 0\times 1 + 1\times \frac{1}{10} + 2\times \frac{1}{100} + 3\times \frac{1}{1000} + 4\times \frac{1}{10000}$

$= 0\times 10^0 + 1\times 10^{-1} + 2\times 10^{-2} + 3\times 10^{-3} + 4\times 10^{-4}$

Exercise 2.2

1. Evaluate :

(a) $\left(\frac{1}{3}\right)^{-4}$

$= \left(\frac{3}{1}\right)^4$

$= (3)^4$

$= 81$

(b) $5^{-1}\times 5^4$

$= (5)^{-1+4}$

$= (5)^3$

$= 125$

(c) $\left[\left(\frac{7}{25}\right)^{-1}\right]^2$

$= \left(\frac{7}{25}\right)^{-1\times 2}$

$= \left(\frac{7}{25}\right)^{-2}$

$= \left(\frac{25}{7}\right)^2$

$= \frac{625}{49}$

(d) $\left\{\left(\frac{1}{3}\right)^{-1}\times(-9)^{-1}\right\}^{-1}$

$= \left\{(3)^1\times\left(\frac{-1}{9}\right)^1\right\}^{-1}$

$= \left\{3\times\frac{-1}{9}\right\}^{-1}$

$= \left(\frac{-1}{3}\right)^{-1}$

$= (-3)^1 = (-3)$

2. Simplify and express the result with positive exponent :

(a) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$= (2)^2 + (3)^2 + (4)^2$

$= 4 + 9 + 16$

$= 29$

(b) $\left[\left(\frac{1}{2}\right)^{-1} - \left(\frac{-3}{4}\right)^{-1}\right]^{-2}$

$= \left[(2)^1 - \left(\frac{-4}{3}\right)^1\right]^{-2}$

$= \left[\frac{2}{1} + \frac{4}{3}\right]^{-2}$

$= \left(\frac{10}{3}\right)^{-2}$

$= \left(\frac{3}{10}\right)^2$

(c) $\left[\left(\frac{3}{8}\right)^{-2}\right]^{3}$

$= \left(\frac{3}{8}\right)^{-2\times 3}$

$= \left(\frac{3}{8}\right)^{-6}$

$= \left(\frac{8}{3}\right)^{6}$

(d) $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$

$= [(3)^3 - (2)^3] \div (4)^3$

$= (27 - 8) \div 64 = 19 \div 64$

$= \frac{19}{64}$

3. Find the value of :

(a) $(2^{-1} + 3^{-1} + 4^{-1})^0$

$= \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)^0$

$= \left(\frac{6+4+3}{12}\right)^0$

$= \left(\frac{13}{12}\right)^0$

$= 1$

(b) $(4^{-1} - 3^{-3} + 6^{-2})^{-1}$

$= \left(\frac{1}{4} - \frac{1}{3^3} + \frac{1}{6^2}\right)^{-1}$

$= \left(\frac{1}{4} - \frac{1}{27} + \frac{1}{36}\right)^{-1}$

$= \left(\frac{27-4+3}{108}\right)^{-1}$

$= \left(\frac{26}{108}\right)^{-1}$

$= \frac{108}{26} = \frac{54}{13}$

(c) $(2^0 + 3^{-1}) \times 9^2$

$= \left(1 + \frac{1}{3}\right) \times 81$

$= \left(\frac{4}{3}\right) \times 81$

$= 108$

(d) $\left[\left(\frac{1}{4}\right)^{-2} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{2}\right)^{-3}$

$= [(4)^2 - (3)^3] \div (2)^3$

$= (16 - 27) \div 8$

$= -11 \div 8 = \frac{-11}{8}$

(e) $[(3)^{-1} \times (4)^{-1}]^{-1} \times 6^{-1}$

$= \left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{6}$

$= \left(\frac{1}{12}\right)^{-1} \times \frac{1}{6}$

$= 12 \times \frac{1}{6}$

$= 2$

(f) $\left(\frac{2}{3}\right)^{-7} \div \left(\frac{2}{3}\right)^{4}$

$= \left(\frac{2}{3}\right)^{-7-4}$

$= \left(\frac{2}{3}\right)^{-11}$

$= \left(\frac{3}{2}\right)^{11}$

4. Let the required number be x

$\therefore \quad (-12)^{-1} \div x = (-4)^{-1}$

or $\quad \left(\frac{-1}{12}\right) \times \frac{1}{x} = \left(\frac{-1}{4}\right)$

or $\quad \frac{1}{x} = \left(\frac{-1}{4}\right) \times \left(\frac{12}{-1}\right)$

or $\quad \frac{1}{x} = 3$

$\therefore \quad x = \frac{1}{3}$

5. Let the required number be x

$\therefore \quad x \times (2)^{-4} = 2^2$

or $\quad x \times \left(\frac{1}{2}\right)^4 = 4$

or $\quad x = 4 \div \left(\frac{1}{2}\right)^4$

$\therefore \quad x = 4 \times (2)^4$

$x = 4 \times 16$

$x = 64$

6. Let the required number be x

$\therefore \quad x \times \left(\frac{2}{7}\right)^{-2} = \left(\frac{5}{7}\right)^{-1}$

or $\quad x = \left(\frac{5}{7}\right)^{-1} \div \left(\frac{2}{7}\right)^{-2}$

$= \left(\frac{7}{5}\right)^{1} \div \left(\frac{7}{2}\right)^{2}$

$= \frac{7}{5} \times \left(\frac{2}{7}\right)^{2}$

$= \frac{7}{5} \times \frac{4}{49} = \frac{4}{35}$

7. Find the value of x in each of the following :

(a) $\left(\frac{2}{3}\right)^{-4} \times \left(\frac{2}{3}\right)^{-8} = \left(\frac{2}{3}\right)^{4x}$

or $$\left(\frac{2}{3}\right)^{-4-8} = \left(\frac{2}{3}\right)^{4x}$$

$$\left(\frac{2}{3}\right)^{-12} = \left(\frac{2}{3}\right)^{4x}$$

Since bases are same, equating the powers,

$$-12 = 4x$$

$$\Rightarrow \quad x = -3$$

(b) $$\left(\frac{1}{3}\right)^{2x} \times \left(\frac{1}{3}\right)^{2} = 3^4$$

$$\left(\frac{1}{3}\right)^{2x+2} = \left(\frac{1}{3}\right)^{-4}$$

Since bases are same, equating the powers,

$$2x + 2 = -4$$

$$\Rightarrow \quad 2x = -4 - 2$$

$$\Rightarrow \quad 2x = -6$$

$$\Rightarrow \quad x = -3$$

(c) $$2^{5x} \div 2^{x} = 2^4$$

$$2^{5x-x} = 2^4$$

$$2^{4x} = 2^4$$

Since bases are same, equating the powers,

$$4x = 4$$

$$\Rightarrow \quad x = 1$$

(d) $$\left(\frac{4}{5}\right)^{3x+1} \times \left(\frac{4}{5}\right)^{-15} = \left(\frac{4}{5}\right)^{x}$$

$$\left(\frac{4}{5}\right)^{3x+1-15} = \left(\frac{4}{5}\right)^{x}$$

$$\left(\frac{4}{5}\right)^{3x-14} = \left(\frac{4}{5}\right)^{x}$$

Since bases are same, equating the powers,

$$3x - 14 = x$$

$$\Rightarrow \quad 3x - x = 14$$

$$\Rightarrow \quad 2x = 14$$

$$\Rightarrow \quad x = 7$$

8. Simplify :

(a) $\dfrac{5^2 \times p^{-4}}{5^3 \times 10 \times p^{-8}}$

$$= \frac{5^2 \times p^8}{5^3 \times 5 \times 2 \times p^4}$$

$$= \frac{5^2 \times p^4}{5^4 \times 2}$$

$$= \frac{1 \times p^4}{5^2 \times 2}$$

$$= \frac{p^4}{25 \times 2}$$

$$= \frac{p^4}{50}$$

(b) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

$$= \frac{5^7 \times 6^5 \times 125}{3^5 \times 10^5}$$

$$= \frac{5^7 \times (2^5 \times 3^5) \times 5^3}{3^5 \times (2^5 \times 5^5)}$$

$$= \frac{5^7 \times 2^5 \times 3^5 \times 5^3}{3^5 \times 2^5 \times 5^5}$$

$$= \frac{5^{10}}{5^5}$$

$$= 5^{10-5}$$

$$= 5^5$$

$$= 3125$$

Exercise 2.3

1. Express the following numbers in standard form :

(a) 242,00,00,00,000

$$= 242 \times 10^9$$

$$= 2.42 \times 10^{9+2} = 2.42 \times 10^{11}$$

(b) 0.00008×10^{-2}

$$= 8.0 \times 10^{-2-5}$$

$$= 8.0 \times 10^{-7}$$

(c) 0.000000029

$$= 2.9 \times 10^{-8}$$

(d) 100.001×10^3

$$= 1.00001 \times 10^{3+2}$$

$$= 1.00001 \times 10^5$$

(e) 30303000000

$$= 30303 \times 10^6$$

$$= 3.0303 \times 10^{6+4}$$

$$= 3.0303 \times 10^{10}$$

2. Express the following in usual form :

(a) 3×10^{-8}

$= 3 \times \frac{1}{100000000}$

$= 0.00000003$

(b) 3.253×10^{-9}

$= 3.253 \times \frac{1}{1000000000}$

$= 0.000000003253$

(c) 1.31×10^{-9}

$= 1.31 \times \frac{1}{1000000000}$

$= 0.00000000131$

(d) 8.37×10^{-6}

$= 8.37 \times \frac{1}{1000000}$

$= 0.00000837$

(e) 6.312×10^{5}

$= 6.312 \times 10.0000$

$= 631200.000$

$= 631200$

3. Mass of a proton 0 $= 1.67 \times 10^{-24}$ g

Mass of an electron $= 9.10 \times 10^{-28}$ g

Ratio of their masses $= \frac{1.67 \times 10^{-24}}{9.10 \times 10^{-28}}$

$= \frac{1.67}{9.10} \times 10^{4}$

$= \frac{16700}{9.10}$

$= \frac{1835}{1}$

$= 1835 : 1$ (approx)

4. Mass of 1 cell $= 1600 \times 10^{-20}$ g

Mass of 200 cells $= 1600 \times 10^{-20}$ g $\times 200$

$= 320000 \times 10^{-20}$ g

$= 3.2 \times 10^{-20+5}$ g

$= 3.2 \times 10^{-15}$ g

5. Express the number which appears in each of the following statements in its standard form :

(a) Speed of light $=$ 30,00,00,000 m/s

$= 3 \times 10^{8}$ m/s

$= 3.0 \times 10^{8}$ m/s

(b) Diameter of a nucleus of an atom = 0.0000000000000075 m

$= 7.5 \times 10^{-15}$ m

(c) Distance of the moon from the earth

= 384,460,000 m

$= 3.8446 \times 10^{8}$ m

(d) Size of an animal cell = 0.0000068 m

$= 6.8 \times 10^{-6}$ m

3. Squares and Square Roots

Exercise 3.1

1. Which of the following cannot be a perfect square? Explain why.

(a) 6937 : It is not a perfect square as the digit at ones place in the number is 7.

(b) 260 : It is a perfect square as the digit at ones place in the number is 0.

(c) 2121 : It is a perfect square as the digit at ones place in the number is 1.

(d) 5298 : It is not a perfect square as the digit at ones place in the number is 8.

2. What will be the digit in ones place of the squares of the following numbers :

(a) 52

unit digit of $(52)^2$

$= (2)^2 = 4$

(b) 83

unit digit of $(83)^2$

$= (3)^2 = 9$

(c) 156

unit digit of $(156)^2$

$= (6)^2 = 36$

$\therefore$ unit digit = 6

(d) 345

unit digit of $(345)^2$

$= (5)^2 = 25$

$\therefore$ unit digit = 5

3. Evaluate the following using properties of perfect squares.

(a) $(29)^2 - (28)^2$

$\therefore \quad (n+1)^2 - n^2 = (n+1) + n$

$(29)^2 - (28)^2 = 29 + 28 = 57$

(b) $(121)^2 - (120)^2$

$\therefore \quad (n+1)^2 - n^2 = (n+1) + n$

$(121)^2 - (120)^2 = 121 + 120 = 241$

(c) $(38)^2 - (37)^2$

$\therefore \quad (n+1)^2 - n^2 = (n+1) + n$

$(38)^2 - (37)^2 = 38 + 37$

$= 75$

4. Without actually adding, find the sum of :

(a) $1+3+5+7+9+11+13+15+17$

Total number = 9

$$\text{Sum} = (9)^2 = 81$$

(b) $1+3+5+7+9+11$

Total number = 6

$$\text{Sum} = (6)^2 = 36$$

5. Observe the following pattern :

We have

$$(7)^2 = 49$$

$$(67)^2 = 4489$$

$$(667)^2 = 444889$$

$$(6667)^2 = 44448889$$

So, $(66667)^2 = 4444488889$

$$(666667)^2 = 444444888889$$

6. Find the Pythagorean triplets of the numbers, the smallest of which is given below :

(a) Number $(2m) = 12$

$\therefore$ $2m = 12$

$$m = \frac{12}{2} = 6$$

$$m = 6$$

$\therefore$ $m^2 - 1 = (6)^2 = 36 - 1 = 35$

$$m^2 + 1 = (6)^2 + 1 = 36 + 1 = 37$$

Thus, the triplets are 12, 35 and 37.

(b) Number = 6

$\therefore$ $2m = 6$

$$m = 3$$

$\therefore$ $m^2 - 1 = (3)^2 - 1 = 9 - 1 = 8$

$$m^2 + 1 = (3)^2 + 1 = 9 + 1 = 10$$

Thus, the triplets are 6, 8 and 10.

(c) Number = 18

$$2m = 18$$

$$m = 9$$

$\therefore$ $m^2 + 1 = (9)^2 + 1 = 81 + 1 = 82$

$$m^2 - 1 = (9)^2 - 1 = 81 - 1 = 80$$

Thus, the triplets are 18, 80 and 82.

(d) Number = 20

$$2m = 20$$

$$m = 10$$

$\therefore$ $m^2 - 1 = (10)^2 - 1 = 100 - 1 = 99$

$$m^2 + 1 = (10)^2 + 1 = 100 + 1 = 101$$

Thus, the triplets are 20, 99 and 101.

7. Indicate whether the squares of the following numbers are even or odd.

(a) 53

Square of $53 = (53)^2$

unit digit $= (3)^2 = 9$

$\therefore$ Square of 53 is odd.

(b) 126

Square of $126 = (126)^2$

unit digit $= (6)^2 = 36$

So, the square of 126 is even.

(c) 289

Square of 289 = $(289)^2$

unit digit = $(9)^2 = 81$

So, the square of 289 is odd.

(d) 2450

Square of 2450 = $(2450)^2$

unit digit = $(0)^2 = 0$

So, the square of 2450 is even.

8. The LCM of 8, 9 and 10 = 3600

$\therefore \quad 3600 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times\overline{5\times 5}$

which is a perfect square.

So, 3600 is the smallest perfect square divisible by 8, 9 and 10.

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

9. Find the smallest number by which the following have to be multiplied to make them a perfect square.

(a) 1792

$\therefore \quad 1792 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times 7$

To make it perfect square, it must be multiplied by 7.

$\therefore \quad 1792\times 7 = 12544$

which is a perfect square.

Hence, 7 is the smallest number by which the given number have to be multiplied to make it a perfect square.

2	1792
2	896
2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

(b) 9075

$\therefore \quad 9075 = 3\times\overline{5\times 5}\times\overline{11\times 11}$

To make it perfect square, it must be multiplied by 3.

$\therefore \quad 9075\times 3 = 27225$

which is a perfect square.

Hence, 3 is the smallest number by which the given number have to be multiplied to make it a perfect square.

3	9075
5	3025
5	605
11	121
11	11
	1

(c) 2028

$\therefore \quad 2028 = \overline{2\times 2}\times 3\times\overline{13\times 13}$

To make it perfect square, it must be multiplied by 3.

$\therefore \quad 2028\times 3 = 6084$

which is a perfect square.

Hence, 3 is the smallest number by which the given number have to be multiplied to make it a perfect square.

2	2028
2	1014
3	507
13	169
13	13
	1

(d) 8575

5	8575
5	1715
7	343
7	49
7	7
	1

$\therefore$ $$8575 = \overline{5 \times 5} \times 7 \times \overline{7 \times 7}$$

To make it perfect square, it must be multiplied by 7.

$$8575 \times 7 = 60025$$

which is a perfect square.

Hence, 7 is the smallest number by which the given number have to be multiplied to make it a perfect square.

10. Find the smallest number by which the following have to be divided to make them a perfect square.

(a) 1734

2	1734
3	867
17	289
17	17
	1

$\therefore$ $$1734 = 2 \times 3 \times \overline{17 \times 17}$$

To make it perfect square, it must be divided by (2×3) 6.

$$1734 \div 6 = 289$$

which is a perfect square.

Hence, 6 is the smallest number by which the given number have to be divided to make it a perfect square.

(b) 2904

2	2904
2	1452
2	726
3	363
11	121
11	11
	1

$\therefore$ $$2904 = \overline{2 \times 2} \times 2 \times 3 \times \overline{11 \times 11}$$

To make it perfect square, it must be divided by (2×3) 6.

$$2904 \div 6 = 484$$

which is a perfect square.

Hence, 6 is the smallest number by which the given number have to be divided to make it a perfect square.

(c) 12150

2	12150
3	6075
3	2025
3	675
3	225
3	75
5	25
5	5
	1

$\therefore$ $$12150 = 2 \times 3 \times \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{5 \times 5}$$

To make it perfect square, it must be divided by (2×3) 6.

$$12150 \div 6 = 2025$$

which is a perfect square.

Hence, 6 is the smallest number by which the given number have to be divided to make it a perfect square.

(d) 14553

3	14553
3	4851
3	1617
7	539
7	77
11	11
	1

$\therefore$ $$14553 = 3 \times \overline{3 \times 3} \times \overline{7 \times 7} \times 11$$

To make it perfect square, it must be divided by (3×11) 33

$$14553 \div 33 = 441$$

which is a perfect square.

Hence, 33 is the smallest number by which the given number have to be divided to make it a perfect square.

11. How many non-square numbers lie between the following pairs of numbers?

(a) $(50)^2$ and $(51)^2$

Non-square numbers between $(n)^2$ and $(n+1)^2 = 2n$.

So, required number $= 2 \times 50 = 100$

(b) $(90)^2$ and $(91)^2$

Non-square numbers between $(n)^2$ and $(n+1)^2 = 2n$.

So, required number $= 2 \times 90 = 180$

12. Find the product of the following :

(a) $53 \times 55 = (54 - 1) \times (54 + 1)$

by $(a + b)(a - b) = a^2 - b^2$

$= (54)^2 - (1)^2 = 2916 - 1 = 2915$

(b) $28 \times 30 = (29 - 1) \times (29 + 1)$

by $(a + b)(a - b) = a^2 - b^2$

$= (29)^2 - (1)^2 = 841 - 1 = 840$

(c) $15 \times 17 = (16 - 1) \times (16 + 1)$

by $(a + b)(a - b) = a^2 - b^2$

$= (16)^2 - (1)^2 = 256 - 1 = 255$

Exercise 3.2

1. Find the square root of the following by prime factorisation :

(a) 729

$729 = \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{3 \times 3}$

$\sqrt{729} = 3 \times 3 \times 3 = 27$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(b) 361

$361 = 19 \times 19$

$\sqrt{361} = 19$

19	361
19	19
	1

(c) 3136

$3136 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{7 \times 7}$

$\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

2	3136
2	1568
2	784
2	392
2	196
2	98
7	49
7	7
	1

(d) 14400

$$14400 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}\times\overline{5\times 5}$$

$$\sqrt{14400} = 2\times 2\times 2\times 3\times 5$$

$$= 120$$

2	14400
2	7200
2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

2. Find the square root of the following by long division method :

(a) 2809

$\sqrt{2809} = 53$

	53
5	$\overline{28}\,\overline{09}$
5	25
103	309
3	309
	×

(b) 2304

$\sqrt{2304} = 48$

	48
4	$\overline{23}\,\overline{04}$
4	16
88	704
8	704
	×

(c) 121

$\sqrt{121} = 11$

	11
1	$\overline{1}\,\overline{21}$
1	1
21	21
1	21
	×

(d) 22201

$\sqrt{22201} = 149$

	149
1	$\overline{2}\,\overline{22}\,\overline{01}$
1	1
24	122
4	96
289	2601
9	2601
	×

3. Find the square root of the following by repeated subtraction method.

(a) 64

1. $64 - 1 = 63$
2. $63 - 3 = 60$
3. $60 - 5 = 55$
4. $55 - 7 = 48$
5. $48 - 9 = 39$
6. $39 - 11 = 28$
7. $28 - 13 = 15$
8. $15 - 15 = 0$

$\therefore \quad \sqrt{64} = 8$

(b) 16

1. $16 - 1 = 15$
2. $15 - 3 = 12$
3. $12 - 5 = 7$
4. $7 - 7 = 0$

$\therefore \quad \sqrt{16} = 4$

(c) 100

1. $100 - 1 = 99$
2. $99 - 3 = 96$
3. $96 - 5 = 91$
4. $91 - 7 = 84$
5. $84 - 9 = 75$
6. $75 - 11 = 64$
7. $64 - 13 = 51$
8. $51 - 15 = 36$
9. $36 - 17 = 19$
10. $19 - 19 = 0$

$\therefore \quad \sqrt{100} = 10$

(d) 144

1. $144 - 1 = 143$
2. $143 - 3 = 140$
3. $140 - 5 = 135$
4. $135 - 7 = 128$
5. $128 - 9 = 119$
6. $119 - 11 = 108$
7. $108 - 13 = 95$
8. $95 - 15 = 80$
9. $80 - 17 = 63$
10. $63 - 19 = 44$
11. $44 - 21 = 23$
12. $23 - 23 = 0$

$\therefore \quad \sqrt{144} = 12$

4. (a) 3450

	58
5	$\overline{34}\,\overline{50}$
5	25
108	950
8	– 864
	86

So, 86 is subtracted from 3450 to get perfect square.

$\therefore \quad 3450 - 86 = 3364$

Hence 3364 is a perfect square.

Now,

$\therefore \quad \sqrt{3364} = 58$

	58
5	$\overline{33}\,\overline{64}$
5	25
108	864
8	864
	×

(b) 7895

	88
8	$\overline{78}\,\overline{95}$
8	64
168	1495
8	1344
	151

So, 151 is subtracted from 7895 to get perfect square.

$$7895 - 151 = 7744$$

Hence 7744 is a perfect square.

Now,

$\therefore$ $\sqrt{7744} = 88$

	88
8	$\overline{77}\,\overline{44}$
8	64
168	1344
8	1344
	×

(c) 27285

	165
1	$\overline{2}\,\overline{72}\,\overline{85}$
1	1
26	172
6	156
325	1685
5	1625
	60

So, 60 is subtracted from 27285 to get perfect square.

$$27285 - 60 = 27225$$

Hence 27225 is a perfect square.

Now,

$\therefore$ $\sqrt{27225} = 165$

	165
1	$\overline{2}\,\overline{72}\,\overline{25}$
1	1
26	172
6	156
325	1625
5	1625
	×

5. (a) 3450

The perfect square is less than $(58)^2$ and next perfect square will be $(59)^2$.

	58
5	$\overline{34}\,\overline{50}$
5	25
108	950
8	864
	86

So, we have $(58)^2 < 3450 < (59)^2$

$$3364 < 3450 < 3481$$

$\therefore$ $3481 - 3450 = 31$

Hence, the required number = 31

31 is the least number which must be added to 3450 to get perfect square.

$\therefore$ $3450 + 31 = 3481$

Now,

$\therefore \qquad \sqrt{3481} = 59$

	59
5	$\overline{34}\,\overline{81}$
5	25
109	981
9	981
	×

(b) 7895

	88
8	$\overline{78}\,\overline{95}$
8	64
168	1495
8	1344
	151

The perfect square is less than $(88)^2$ and next perfect square will be $(89)^2$.

So, we have $\qquad (88)^2 < 7895 < (89)^2$

$$7744 < 7895 < 7921$$

$\therefore \qquad 7921 - 7895 = 26$

Hence, the required number = 26

26 is the least number which must be added to 7895 to get perfect square.

$\therefore \qquad 7895 + 26 = 7921$

Now,

$\therefore \qquad \sqrt{7921} = 89$

	89
8	$\overline{79}\,\overline{21}$
8	64
169	1521
9	1521
	×

(c) 54725

	233
2	$\overline{5}\,\overline{47}\,\overline{25}$
2	4
43	147
3	129
463	1825
3	1389
	436

The perfect square is less than $(233)^2$ and next perfect square will be $(234)^2$.

So, we have $\qquad (233)^2 < 54725 < (234)^2$

$$54289 < 54725 < 54756$$

$\therefore \quad 54756 - 54725 = 31$

Hence, the required number = 31

31 is the least number which must be added to 54725 to get perfect square.

$\therefore \quad 54725 + 31 = 54756$

Now,

$\therefore \quad \sqrt{54756} = 234$

	234
2	$\overline{5}\,\overline{47}\,\overline{56}$
2	4
43	147
3	129
464	1856
4	1856
	×

6.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

$\therefore \quad 9408 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times 3 \times \overline{7 \times 7}$

In the prime factors of 9408, 3 is unpaired.

So, 3 is the smallest number by which 9408 must be divided to get a perfect square.

$9408 \div 3 = 3136$

Now,

$\therefore \quad \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

2	3136
2	1568
2	784
2	392
2	196
2	98
7	49
7	7
	1

7.

2	4802
7	2401
7	343
7	49
7	7
	1

$\therefore \quad 4802 = 2 \times \overline{7 \times 7} \times \overline{7 \times 7}$

In the prime factors of 480, 2 is unpaired.

So, 2 is the smallest number by which 4802 should be multiplied to 4802 to get a perfect square.

$$4802 \times 2 = 9604$$

Now,

$\therefore \quad \sqrt{9604} = 2 \times 7 \times 7 = 98$

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

8. The smallest 4-digit number is 1000.

	31
3	$\overline{1}\,\overline{000}$
3	9
61	100
1	61
	39

We note that 1000 is not a perfect square. The perfect square greater than 1000 is $(32)^2 = 1024$.

Hence, the smallest 4-digit number which is a perfect square is 1024.

9. To get the smallest number which is divisible by 6, 18 and 30, we find the LCM of 6, 18 and 30.

LCM of 6, 18 and 20 = 90

2	6, 18, 30
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

$\therefore \quad 90 = 2 \times \overline{3 \times 3} \times 5$

Clearly, 90 is not a perfect square, so (2×5) 10 is the smallest number by which 90 is to be multiplied to make it perfect square.

$$90 \times 10 = 900$$

and, $\sqrt{900} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}$

$= 2 \times 3 \times 5 = 30$

2	90
3	45
3	15
5	5
	1

10. Let the number of students in the class be x.

Since each student donated as many rupees as there are students in the class, the amount donated by each student = x.
Total amount = $x \times x = 1521$

$$x^2 = 1521$$

$$x = \sqrt{1521}$$

$$= \sqrt{3 \times 3 \times 13 \times 13} = 3 \times 13 = 39$$

Hence, the number of students in the class = 39

Exercise 3.3

1. Find the square root of the following :

(a) $\frac{121}{169}$

11	121
11	11
	1

13	169
13	13
	1

$\therefore \quad \sqrt{\frac{121}{169}} = \sqrt{\frac{11 \times 11}{13 \times 13}} = \frac{11}{13}$

(b) $\frac{64}{441}$

$\therefore \quad \sqrt{\frac{64}{441}} = \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 7 \times 7}}$

$= \frac{2 \times 2 \times 2}{3 \times 7}$

$= \frac{8}{21}$

2	64
2	32
2	16
2	8
2	4
2	2
	1

3	441
3	147
7	49
7	7
	1

(c) $\frac{169}{8100}$

$\therefore \quad \sqrt{\frac{169}{8100}}$

$= \sqrt{\frac{13 \times 13}{2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5}}$

$= \frac{13}{2 \times 3 \times 3 \times 5}$

$= \frac{13}{90}$

13	169
13	13
	1

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
5	5
	1

(d) $\dfrac{100}{361}$

$\therefore \quad \sqrt{\dfrac{100}{361}}$

2	100
2	50
5	25
5	5
	1

19	361
19	19
	1

$$= \sqrt{\frac{2\times 2\times 5\times 5}{19\times 19}}$$

$$= \frac{2\times 5}{19} = \frac{10}{19}$$

2. Find the value of the following :

(a) $\sqrt{5\dfrac{19}{25}}$

2	144
2	72
2	36
2	18
3	9
3	3
	1

5	25
5	5
	1

$$= \sqrt{\frac{144}{25}}$$

$$= \frac{2\times 2\times 3}{5}$$

$$= \frac{12}{5}$$

$$= 2\frac{2}{5}$$

(b) $\sqrt{4\dfrac{29}{49}} = \sqrt{\dfrac{225}{49}}$

3	225
3	75
5	25
5	5
	1

7	49
7	7
	1

$$= \frac{3\times 5}{7}$$

$$= \frac{15}{7}$$

$$= 2\frac{1}{7}$$

(c) $\sqrt{2\dfrac{7}{9}} = \sqrt{\dfrac{25}{9}}$

5	25
5	5
	1

3	9
3	3
	1

$$= \frac{5}{3}$$

$$= 1\frac{2}{3}$$

(d) $\sqrt{6\dfrac{30}{289}} = \sqrt{\dfrac{1764}{289}}$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

17	289
17	17
	1

$$= \frac{2\times 3\times 7}{17}$$

$$= \frac{42}{17}$$

$$= 2\frac{8}{17}$$

3. Find the square root of the following decimals :

(a) $33.64 = \frac{3364}{100}$

2	3364
2	1682
29	841
29	29
	1

2	100
2	50
5	25
5	5
	1

$\therefore \quad \sqrt{\frac{3364}{100}} = \sqrt{\frac{2\times2\times29\times29}{2\times2\times5\times5}}$

$= \frac{2\times29}{2\times5}$

$= \frac{58}{10} = 5.80$

(b) $28.09 = \frac{2809}{100}$

53	2809
53	53
	1

2	100
2	50
5	25
5	5
	1

$\therefore \quad \sqrt{\frac{2809}{100}} = \sqrt{\frac{53\times53}{2\times2\times5\times5}}$

$= \frac{53}{2\times5} = \frac{53}{10} = 5.30$

(c) $4.41 = \frac{441}{100}$

3	441
3	147
7	49
7	7
	1

2	100
2	50
5	25
5	5
	1

$\therefore \quad \sqrt{\frac{441}{100}} = \sqrt{\frac{3\times3\times7\times7}{2\times2\times5\times5}} = \frac{3\times7}{2\times5}$

$= \frac{21}{10} = 2.10$

(d) 3

$\therefore \quad \sqrt{3} = 1.73$ (approx)

	1.732
1	$\overline{3}.\overline{00}\,\overline{00}\,\overline{00}$
1	1
27	200
7	189
343	1100
3	1029
3462	7100
2	6924
	176

4. Find the square root of the following to the nearest integer :

(a) 350

As, $\quad (18)^2 = 324$ and $(19)^2 = 361$

Since, $\quad 324 < 350 < 361$

$18 < \sqrt{350} < 19$

But, 350 is closer to 361 than 324, so, $\sqrt{350}$ to the nearest integer = 19

$\therefore \quad \sqrt{350} = 19$ (approx)

(b) 158

As, $(12)^2 = 144$ and $(13)^2 = 169$

Since, $144 < 158 < 169$

$12 < \sqrt{158} < 13$

But, 158 is closer to 169 than 144, so, $\sqrt{158}$ to the nearest integer $= 13$

$\therefore \quad \sqrt{158} = 13$ (approx)

(c) 645

As, $(25)^2 = 625$ and $(26)^2 = 676$

Since, $625 < 645 < 676$

$25 < \sqrt{645} < 26$

But, 645 is closer to 676 than 625, so, $\sqrt{645}$ to the nearest integer = 26

$\therefore \quad \sqrt{645} = 26$ (approx)

(d) 2925

As, $(54)^2 = 2916$ and $(55)^2 = 3025$

Since, $2916 < 2925 < 3025$

$54 < \sqrt{2925} < 55$

But, 2925 is closer to 3025 than 2916.

So, $\sqrt{2925}$ to the nearest integer = 55

$\therefore \quad \sqrt{2925} = 55$ (approx)

5. Area of the square field $= 132.25\ \text{km}^2$

Area of square = side × side

$132.25 = (\text{side})^2$

$\text{side} = \sqrt{132.25}$

$\text{side} = 11.5\ \text{km}$

$\therefore$ Perimeter of square field = 4×11.5 km = 46.0 km

$\therefore$ Distance travelled by the man in 1 round = 46 km

Hence, the distance travelled by the man in three rounds = 3×46 km = 138 km

4. Cubes & Cube Roots

Exercise 4.1

1. Find the cube of the following :

(a) 25

Cube of $25 = (25)^3$

$= 25 \times 25 \times 25$

$= 15625$

(b) 30

Cube of $30 = (30)^3$

$= 30 \times 30 \times 30$

$= 27000$

(c) 17

Cube of 17 $= (17)^3$

$= 17 \times 17 \times 17$

$= 4913$

(d) 42

Cube of 42 $= (42)^3$

$= 42 \times 42 \times 42$

$= 74088$

2. Find the cube of the following :

(a) -21

$\therefore \quad (-21)^3 = (-21) \times (-21) \times (-21)$

$= -9261$

(b) 0.05

$\therefore \quad (0.05)^3 = 0.05 \times 0.05 \times 0.05$

$= 0.000125$

(c) $\frac{-13}{18}$

$$\therefore \left(\frac{-13}{18}\right)^3 = \left(\frac{-13}{18}\right) \times \left(\frac{-13}{18}\right) \times \left(\frac{-13}{18}\right)$$

$$= \frac{-2197}{5832}$$

(d) $2\frac{3}{11} = \frac{25}{11}$

$$\therefore \left(\frac{25}{11}\right)^3 = \frac{25}{11} \times \frac{25}{11} \times \frac{25}{11}$$

$$= \frac{15625}{1331}$$

3. (a) 4608

$\therefore \ 4608 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times 3 \times 3$

The factors 3×3 do not appear in group of triplet.
If we multiply 4608 by 3, the number obtained will be $4608 \times 3 = 13824$ which is a perfect cube.

So, the required smallest number is 3.

2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(b) 10584

$\therefore \quad 10584 = \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times 7 \times 7$

The factors 7×7 do not appear in group of triplet.

If we multiply 10584 by 7, the number obtained will be $10584 \times 7 = 74088$
which is a perfect cube.

So, the required smallest number is 7.

2	10584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	7
	1

(c) 26244

$\therefore 26244 = 2 \times 2 \times 3 \times 3 \times \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}$

The factors 2×2 do not appear in group of triplet.

If we multiply 26244 by $2 \times 3 = 6$, the number obtained will be 157464 which is a perfect cube.

So, the required smallest number is 6.

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

4. What is the smallest number by which the following numbers must be divided so that the quotient is a perfect cube?

(a) 4374

2	4374
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore$ $$4374 = 2 \times \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3} \times 3$$

Since, the factors 2 and 3 are not grouped in triplets,

so, $2 \times 3 = 6$ is the smallest number by which we divide 4374, the quotient will be a perfect cube.

$\therefore$ $$4374 \div 6 = 729$$

Hence, the smallest number is 6.

(b) 9408

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

$\therefore$ $$9408 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times 3 \times 7 \times 7$$

Since, the factors 3 and 7×7 are not grouped in triplets,

so, $3 \times 7 \times 7 = 147$ is the smallest number by which we divide 9408, the quotient will be a perfect cube.

$\therefore$ $$9408 \div 147 = 64$$

Hence, the smallest number is 147.

(c) 20736

2	20736
2	10368
2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$\therefore$ $$20736 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times 2 \times 2 \times \overline{3 \times 3 \times 3} \times 3$$

Since, the factors 2×2 and 3 are not grouped in triplets,

so, $2 \times 2 \times 3 = 12$ is the smallest number by which we divide 20736, the quotient will be a perfect cube.

$\therefore$ $$20736 \div 12 = 1728$$

Hence, the smallest number is 12.

5. Edge of cube $= 3.8\,\text{cm}$

Volume of cube $= (\text{edge})^2 = (3.8\,\text{cm})^3$

$$= 54.872\,\text{cm}^3$$

Hence, the volume of cube is $54.872\,\text{cm}^3$

6. Express the following cubes as the sum of consecutive odd numbers :

(a) 7^3

$\therefore$ $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$ (Sum of consecutive odd numbers)

(b) 8^3

$\therefore$ $8^3 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$ (Sum of consecutive odd numbers)

(c) 10^3

$\therefore$ $10^3 = 91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109$

(Sum of consecutive odd numbers)

7. (a) 6^3

$\therefore$ $6^3 = 1 + 7 + 19 + 37 + 61 + 91$ (Sum of odd prime numbers)

(b) 7^3

$\therefore$ $7^3 = 1 + 7 + 19 + 37 + 61 + 91 + 127$ (Sum of odd prime numbers)

(c) 8^3

$\therefore$ $8^3 = 1 + 7 + 19 + 37 + 61 + 91 + 127 + 169$ (Sum of odd prime numbers)

8. The value of :

(a) $11^3 - 10^3 = 1 + 11 \times 10 \times 3$

(b) $19^3 - 18^3 = 1 + 19 \times 18 \times 3$

Exercise 4.2

1. Find the cube root of the following by successive subtraction :

(a) 343

1. $343 - 1 = 342$
2. $342 - 7 = 335$
3. $335 - 19 = 316$
4. $316 - 37 = 279$
5. $279 - 61 = 218$
6. $218 - 91 = 127$
7. $127 - 127 = 0$

$\therefore$ $\sqrt[3]{343} = 7$

(b) 512

1. $512 - 1 = 511$
2. $511 - 7 = 504$
3. $504 - 19 = 485$
4. $485 - 37 = 448$
5. $448 - 61 = 387$
6. $387 - 91 = 296$
7. $296 - 127 = 169$
8. $169 - 169 = 0$

$\therefore$ $\sqrt[3]{512} = 8$

(c) 216

1. $216 - 1 = 215$
2. $215 - 7 = 208$
3. $208 - 19 = 189$
4. $189 - 37 = 152$
5. $152 - 61 = 91$
6. $91 - 91 = 0$

$\therefore$ $\sqrt[3]{216} = 6$

2. Find the cube root of the following by prime factorisation method :

(a) 15625

$$\therefore \quad \sqrt[3]{15625} = \sqrt[3]{\overline{5\times5\times5}\times\overline{5\times5\times5}}$$

$$= 5\times5 = 25$$

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(b) 13824

$$\therefore \quad \sqrt[3]{13824} = \sqrt[3]{\overline{2\times2\times2}\times\overline{2\times2\times2}\times\overline{2\times2\times2}\times\overline{3\times3\times3}}$$

$$= 2\times2\times2\times3 = 24$$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(c) –5832

$$\therefore \quad \sqrt[3]{-5832} = -\sqrt[3]{(\overline{2\times2\times2}\times\overline{3\times3\times3}\times\overline{3\times3\times3})}$$

$$= -2\times3\times3 = -18$$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

3. Find the cube root of the following :

(a) $\frac{1331}{1728}$

11	1331
11	121
11	11
	1

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$$\therefore \quad \sqrt[3]{\frac{1331}{1728}} = \sqrt[3]{\frac{11\times11\times11}{2\times2\times2\times2\times2\times2\times3\times3\times3}}$$

$$= \frac{11}{2\times2\times3} = \frac{11}{12}$$

(b) $\frac{-343}{2197}$

7	343
7	49
7	7
	1

13	2197
13	169
13	13
	1

∴ $-\sqrt[3]{\frac{7\times 7\times 7}{13\times 13\times 13}}$

$= \frac{-7}{13}$

(c) $9.261 = \frac{9261}{1000}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

∴ $\sqrt[3]{\frac{9261}{1000}} = \sqrt[3]{\frac{3\times 3\times 3\times 7\times 7\times 7}{2\times 2\times 2\times 5\times 5\times 5}}$

$= \frac{3\times 7}{2\times 5}$

$= \frac{21}{10} = 2.1$

4. Find the cube root of the following :

(a) 144×96

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

∴ $\sqrt[3]{144\times 96}$

$= \sqrt[3]{2\times 2\times 2\times 2\times 3\times 3\times 2\times 2\times 2\times 2\times 2\times 3}$

$= 2\times 2\times 2\times 3$

$= 24$

(b) 45×75

3	45
3	15
5	5
	1

3	75
5	25
5	5
	1

∴ $\sqrt[3]{45\times 75}$

$= \sqrt[3]{3\times 3\times 5\times 3\times 5\times 5}$

$= 3\times 5$

$= 15$

(c) -216×729

2	216
2	108
2	54
3	27
3	9
3	3
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

∴ $-\sqrt[3]{216\times 729}$

$= -\sqrt[3]{2\times 2\times 2\times 3\times 3\times 3\times 3\times 3\times 3\times 3\times 3\times 3}$

$= -2\times 3\times 3\times 3 = -54$

5.

2	4116
2	2058
3	1029
7	343
7	49
7	7
	1

$\therefore$ $4116 = 2 \times 2 \times 3 \times \overline{7 \times 7 \times 7}$

Making groups of triplets, we see that the factors 2×2 and 3 are not in triplets.

Hence, if we multiply 4116 by $2 \times 3 \times 3 = 18$, we get

$$4116 \times 18 = 74088$$

So, 18 is the smallest number.

Now,

$\therefore$ $\sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

6.

2	250
5	125
5	25
5	5
	1

$\therefore$ $250 = 2 \times \overline{5 \times 5 \times 5}$

As factor 2 does not form a triple, dividing 250 by 2 will give us a perfect cube.

$\therefore$ $250 \div 2 = 125$

and $125 = 5 \times 5 \times 5$

$\sqrt[3]{125} = 5$

7. Volume of metallic cube = 343 cm^3

$\therefore$ Volume of cube = $(\text{edge})^3$

$343 = (\text{edge})^3$

$\text{edge} = \sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7\ \text{cm}$

Hence, the edge of cube is 7 cm.

5. Playing with Numbers

Exercise 5.1

1. (a) We know that $ab + ba = 11(a + b)$

here $49 + 94 = 11 \times (9 + 4)$

$= 11 \times 13$

(i) When the sum is divided by 11, the quotient is 13.

(ii) When the sum is divided by 13, the quotient is 11.

(b) We have, $ab + ba = 11\,(a + b)$

$$54 + 45 = 11 \times (4 + 5)$$
$$= 11 \times 9$$

(i) When the sum is divided by 11, the quotient is 9.

(ii) When the sum is divided by 9, the quotient is 11.

2. We have, $abc + bca + cab = 11 \times (a + b + c)$

$$637 + 376 + 763 = 111 \times (3 + 6 + 7)$$
$$= 111 \times 16 = 3 \times 37 \times 16$$

(i) When the sum is divided by 111, the quotient is 16.

(ii) When the sum is divided by 16, the quotient is 111.

(iii) When the sum is divided by 37, the quotient is $3 \times 16 = 48$.

3. We have $(ab - ba) = 9\,(a - b)$

$$93 - 39 = 9 \times (9 - 3)$$
$$= 9 \times 6 = 9 \times 3 \times 2 = 27 \times 2$$

(i) When the difference is divided by 9, the quotient is 6.

(ii) When the difference is divided by 27, the quotient is 2.

4. (a) 99 (b) 5 (c) 33

We have $abc - cba = 99\,(a - c)$

$$863 - 368 = 99\,(8 - 3)$$
$$= 99 \times 5 = 3 \times 33 \times 5$$

(a) When the difference is divided by 99, the quotient is 5.

(b) When the difference is divided by 5, the quotient is 99.

(c) When the difference is divided by 33, the quotient is $3 \times 5 = 15$.

5. We have $abc - cba = 99\,(a - c)$

$$892 - 298 = 99\,(8 - 2)$$
$$= 99 \times 6 = 11 \times 9 \times 6$$
$$= 11 \times 3 \times 3 \times 6 = 33 \times 3 \times 6$$

(i) When the difference is divided by 6, the quotient is 99.

(ii) When the difference is divided by 9, the quotient is $11 \times 6 = 66$.

(iii) When the difference is divided by 33, the quotient is $3 \times 6 = 18$.

Exercise 5.2

1. Sum of the digits $31a2 = 3 + 1 + a + 2$
$= 6 + a$

The given number is divisible by 9. If the sum of digits is divisible by 9, then $6 + a$ is divisible by 9.

$\therefore \quad 6 + a = 9K \quad [K = 1]$

$\Rightarrow \quad a = 9 - 6$

$\Rightarrow \quad a = 3$

2. Sum of the digits $4n7 = 4 + n + 7 = 11 + n$
If the given number is divisible by 3, then $11 + n$ is divisible by 9.

$\therefore \quad 11 + n = 3K \quad [K = 4]$

$11 + n = 3 \times 4$

$11 + n = 12$

$\Rightarrow \quad n = 12 - 11$

$\Rightarrow \quad n = 1$

3. The number $814y$ is divisible by 6.

$\Rightarrow$ $814y$ is divisible by both 2 and 3.

$\therefore$ $814y$ is divisible by 2.

$\Rightarrow$ y is an even digit

$\Rightarrow \quad y = 0, 2, 4, 6, 8 \quad$ ...(1)

$\therefore$ $814y$ is divisible by 3.

$\Rightarrow \quad 8 + 1 + 4 + y = 0, 3, 6, 9, 12, 15, \ldots\ldots$

$\Rightarrow \quad 13 + y = 0, 3, 6, 9, 12, 15, \ldots\ldots \quad$ (2)

From (1), we have

$n = 0, 2, 4, 6, 8$

$\Rightarrow \quad 13 + y = 13, 15, 17, 19, 21 \quad$ ...(3)

From (2) and (3)

$13 + y = 15 \text{ or } 21$

$\Rightarrow \quad y = 2 \text{ or } 8$

The required values of y is 2 or 8.

4. Replace b by the smallest possible digit so that $123b$ is divisible by :

(a) 2 (b) 3 (c) 10 (d) 5

(a) The number $123b$ is divisible by 2.

$\Rightarrow$ b is an even digit.

$\Rightarrow \qquad b = 0, 2, 4, 6, 8$

So, 0 is the smallest possible digit so that $123b$ is divisible by 2.

$\therefore \qquad b = 0$

(b) The number $123b$ is divisible by 3.

Sum of digits $= 1 + 2 + 3 + b = 6 + b$

$6 + b$ is divisible by 3

$\therefore \qquad 6 + b = 3K \qquad [K = 2]$

$6 + b = 6$

$b = 6 - 6$

$b = 0$

So, the smallest number is 0 to make the number $123b$ divisible by 3.

(c) The number $123b$ is divisible by 10.

$\Rightarrow \qquad b$ must be 0

So, the smallest number is 0 to make the number $123b$ divisible by 10.

(d) The number $123b$ is divisible by 5.

$\Rightarrow \qquad b$ must be 0 or 5

So, the smallest number is 0 to make the number $123b$ divisible by 5.

$\therefore \qquad b = 0$

5. (a) 3, 75, 273

The units digit of the given number is 3 which is not divisible by 5, as it is less than 5. So, the required remainder is 3.

Now, when we divide 3 (units digit) by 2, the remainder is 1.

So, the required remainder is 1.

(b) 78,562

The sum of digits of the number $= 7 + 8 + 5 + 6 + 2 = 28$

The remainder is 1 when 28 is divided by 3.

So, the required remainder is 1.

Now, the number formed by taking the tens digit and units digit of the given number is 62. When 62 is divided by 4, the remainder is 2. So, the required remainder is 2.

(c) 3,45,067

The units digit of the given number is 7. When 7 is divided by 5, the remainder is 2. So, the required remainder is 2. Now, the units digit of the given number is 7, and it is less than 10.

So, the required remainder is 7.

(d) 48,962

The sum of digits of the number = $4 + 8 + 9 + 6 + 2 = 29$

The remainder is 2 when 29 is divided by 3.

So, the required remainder is 2.

Now, the sum of digits of the number = 29

The remainder is 2 when 29 is divided by 9.

So, the required remainder is 2.

Exercise 5.3

1. (a)

$$\begin{array}{rrr} & 4 & A \\ + & 7 & 9 \\ \hline C & B & 5 \\ \hline \end{array}$$

We have $9 + A = 5$

Put $A = 6$, we get $9 + 6 = 15$ having 5 as units place.

Now, $1 + 4 + 7 = B$

$B = 12$

$\Rightarrow$ $CB = 12$

$\therefore$ $A = 6,\quad B = 2,\quad C = 1$

(b)

$$\begin{array}{rrr} & A & B \\ + & 3 & 7 \\ \hline & 6 & A \\ \hline \end{array}$$

Here, $B = 5$

$\Rightarrow$ $7 + 5 = 12$

$\therefore$ $A = 2$

Now, $1 + A + 3 = 6$

$A = 6 - 4$

$A = 2$

So, $A = 2$ and $B = 5$

(c)

$$\begin{array}{rrrr} & Q & 2 & 4 \\ + & Q & P & 6 \\ \hline 1 & P & 3 & P \\ \hline \end{array}$$

Here, $4 + 6 = 10$

$\therefore$ $P = 0$

Now, $1 + 2 + P = 3$

$\Rightarrow$ $P = 0$

and $Q + Q = 1P$

$$2Q = 10$$

$$Q = \frac{10}{2}$$

$$Q = 5$$

$\therefore$ $P = 0, \quad Q = 5$

(d)
$$\begin{array}{r} 2\ A\ B \\ +\ 5\ 9\ B \\ \hline A\ 7\ A \\ \hline \end{array}$$

Here, $B + B = A$

$$2B = A \quad \ldots(1)$$

and $A + 9 = 7$

$\therefore$ $A = 8$

$8 + 9 = 17$ (having 7 as units place)

(f) $2B = 8$

$$B = \frac{8}{2} = 4$$

$1 + 2 + 5 = A$ $\quad A = 8$

$\therefore$ $A = 8, \quad B = 4$

(e)
$$\begin{array}{r} A\ 8\ 4\ B \\ -\ C\ 8\ 1 \\ \hline A\ 0\ A\ 9 \\ \hline \end{array}$$

Here, $B = 10$ (having 0 as units place)

then $13 - 8 = A$

$\Rightarrow$ $A = 5$

$\therefore$ $8 - C = 0$

$$C = 8$$

$\therefore$ $A = 5, \quad B = 0, \quad C = 8$

(f)
$$\begin{array}{r} A\ 9 \\ \times\ B \\ \hline A\ A\ 1 \\ \hline \end{array}$$

Here, $B = 9$

$\therefore$ $9 \times 9 = 81$

Now, $9 \times A + 8 = AA$

Put $A = 4$

$$36 + 8 = AA$$

$$44 = AA$$

$\therefore$ $A = 4,\quad B = 9$

(g)

$$\begin{array}{ccc} & A & B \\ & \times & 9 \\ \hline 6 & A & B \\ \hline \end{array}$$

Here, $B \times 9 = B$

Put $B = 5$

$5 \times 9 = 45$ (having 5 as units place)

Now, $9 \times A + 4 = 6A$

Put $A = 7$

$$63 + 4 = 6A$$

$$67 = 6A$$

$$A = 7$$

$\therefore$ $A = 7,\quad B = 5$

6. Algebraic Expressions

Exercise 6.1

1. Write the terms and their numerical coefficients for each of the following expressions.

(a) $2ab^2 - 3ab$

Terms $\rightarrow 2ab^2, -3ab$

Coefficient of $ab^2 = 2$

Coefficient of ab in $-3ab = -3$

(b) $-5xy + 3y^2z$

Terms $\rightarrow -5xy, 3y^2z$

Coefficient of xy in $-5xy = -5$

Coefficient of y^2z in $3y^2z = 3$

(c) $m^2 - m - 1$

Terms $\rightarrow m^2, -m, -1$

Coefficient of m^2 in $m^2 = 1$

Coefficient of m in $-m = -1$

(d) $\frac{5}{4}x^2y - \frac{3}{2}xy^2$

Terms $\rightarrow \frac{5}{4}x^2y,\ -\frac{3}{2}xy^2$

Coefficient of x^2y in $\frac{5}{4}x^2y = \frac{5}{4}$

Coefficient of xy^2 in $\frac{-3}{2}xy^2 = \frac{-3}{2}$

(e) $-a - 2b + c + 6$

Terms $\rightarrow -a, -2b, c, 6$

Coefficient of a in $-a = -1$

Cofficient of b in $-2b = -2$

Coefficient of c in $c = 1$

2. Identify the like terms in the following expressions :

(a) $-112x^3 - 3x^2 + 5x^3$

In the given expression $-112x^3, 5x^3$ are like terms.

(b) $72a + 13b - 16a$

In the given expression $72a$ and $-16a$ are like terms.

(c) $15xy + 3x + 2y - 3xy$

In the given expression $15xy$ and $-3xy$ are like terms.

(d) $12x^2 + 5x + 9$

In the given expression, there are no like terms because there are no terms having same variable.

(e) $315pq - 45q^2 + 5pq$

In the given expression $315pq$ and $5pq$ are like terms.

3. Add the following :

(a) $5a^2 - 3b^2, 16b^2 - 3a^2$ and $-11a^2 - 9b^2$

$$= (5a^2 - 3b^2) + (16b^2 - 3a^2) + (-11a^2 - 9b^2)$$

$$= (5a^2 - 3a^2 - 11a^2) + (-3b^2 + 16b^2 - 9b^2)$$

$$= (2a^2 - 11a^2) + (13b^2 - 9b^2)$$

$$= -9a^2 + 4b^2$$

(b) $4p^3 + 3pq + 3$ and $2p^3 - 6pq + 11$

$$= (4p^3 + 3pq + 3) + (2p^3 - 6pq + 11)$$

$$= (4p^3 + 2p^3) + (3pq - 6pq) + (3 + 11)$$

$$= 6p^3 - (3pq) + 14$$

$$= 6p^3 - 3pq + 14$$

(c) $8x^2 + 7x + 12, 17x^2 - 15x - 21$ and $4 - 9x^2 + 11x$

$$= (8x^2 + 7x + 12) + (17x^2 - 15x - 21) + (4 - 9x^2 + 11x)$$

$= (8x^2 + 17x^2 - 9x^2) + (7x - 15x + 11x) + (12 - 21 + 4)$

$= (25x^2 - 9x^2) + (-8x + 11x) + (-9 + 4)$

$= 16x^2 + 3x - 5$

(d) $15m^2n - 17mn + 8mn^2, 10m^2n - 12mn - 9mn^2$ and $12m^2n + 21mn - 14mn^2$

$= (15m^2n - 17mn + 8mn^2) + (10m^2n - 12mn - 9mn^2) + (12m^2n + 21mn - 14mn^2)$

$= (15m^2n + 10m^2n + 12m^2n) + (-17mn - 12mn + 21mn)\ (8mn^2 - 9mn^2 - 14mn^2)$

$= (25m^2n + 12m^2n) + (-29mn + 21mn) + (-mn^2 - 14mn^2)$

$= 37m^2n - 8mn - 15mn^2$

(e) $11a + 15b - 8c + 7d, 10a + 9c - 13d$ and $9a + 8b - 11c - 17d$

$= (11a + 15b - 8c + 7d) + (10a + 9c - 13d) + (9a + 8b - 11c - 17d)$

$= (11a + 10a + 9a) + (15b + 8b) + (-8c + 9c - 11c) + (7d - 13d - 17d)$

$= (21a + 9a) + (23b) + (c - 11c) + (-6d - 17d)$

$= 30a + 23b - 10c - 23d$

(f) $\frac{5}{2}x^2 - \frac{3}{5}y, \frac{-11}{4}x^2 - \frac{13}{10}y$ and $\frac{-1}{2}y + \frac{3}{4}x^2$

$= \left(\frac{5}{2}x^2 - \frac{3}{5}y\right) + \left(\frac{-11}{4}x^2 - \frac{13}{10}y\right) + \left(\frac{-1}{2}y + \frac{3}{4}x^2\right)$

$= \left(\frac{5}{2}x^2 - \frac{11}{4}x^2 + \frac{3}{4}x^2\right) + \left(\frac{-3}{5}y - \frac{13}{10}y - \frac{1}{2}y\right)$

$= \frac{1}{2}x^2 + \left(\frac{-12}{5}y\right)$

$= \frac{1}{2}x^2 - \frac{12}{5}y$

4. Subtract the following :

(a) $16p^2q$ from $-11p^2q$

$= -11p^2q - 16p^2q$

$= -27p^2q$

(b) $x^2 + 3a + 4$ from $4x^2 - 6a - 1$

$= (4x^2 - 6a - 1) - (x^2 + 3a + 4)$

$= (4x^2 - 6a - 1 - x^2 - 3a - 4)$

$= (4x^2 - x^2) + (-6a - 3a) + (-1 - 4)$

$= 3x^2 - 9a - 5$

(c) $25abc + 15a^3$ from $-15bca - 10$

$= (-15bca - 10) - (25abc + 15a^3)$

$= -15bca - 10 - 25abc - 15a^3$

$= (-15bca - 25abc) + (-15a^3) - 10$

$= -40abc - 15a^3 - 10$

(d) $21ax + 15by - 6cz$ from $12ax + 6by + 11cz$

$= (12ax + 6by + 11cz) - (21ax + 15by - 6cz)$

$= (12ax - 21ax) + (6by - 15by) + (11cz + 6cz)$

$= -9ax - 9by + 17cz$

(e) $16x^3 + 14x^2 - 9x + 15$ from $17x^3 - 19x^2 + 13x - 21$

$= (16x^3 + 14x^2 - 9x + 15) - (17x^3 - 19x^2 + 13x - 21)$

$= (16x^3 - 17x^3) + (14x^2 + 19x^2) + (-9x - 13x) + (15 + 21)$

$= -x^3 + 33x^2 - 22x + 36$

5. Let the required expression be X.

$(23x^2 - 11xy + 15y^2) + X = 22xy - 3x^2 - 5y^2$

$X = (22xy - 3x^2 - 5y^2) - (23x^2 - 11xy + 15y^2)$

$= (22xy + 11xy) + (-3x^2 - 23x^2) + (-5y^2 - 15y^2)$

$= 33xy - 26x^2 - 20y^2$

6. Sum of $11p + 9q - 7r$ and $-7p - 12q + 8r$

$= (11p + 9q - 7r) + (-7p - 12q + 8r)$

$= (11p - 7p) + (9q - 12q) + (-7r + 8r)$

$= 4p - 3q + r$

Sum of $6p + 14q - 13r$ and $15p - q + 12r$

$= (6p + 14q - 13r) + (15p - q + 12r)$

$= (6p + 15p) + (14q - q) + (-13r + 12r)$

$= 21p + 13q - r$

Now, subtract $21p + 13q - r$ from $4p - 3q + r$

$= (4p - 3q + r) - (21p + 13q - r)$

$= (4p - 21p) + (-3q - 13q) + (r + r)$

$= -17p - 16q + 2r$

7. Perimeter of a triangle = Sum of all sides

$$7x^2 + 7x - 7 = (5x + 56) + (-16x^2 - 3x - 1) + c$$

$$7x^2 + 7x - 7 = (5x - 3x) - 16x^2 + (56 - 1) + c$$

$$7x^2 + 7x - 7 = 2x - 16x^2 + 55 + c$$

$$c = 7x^2 + 7x - 7 - 2x + 16x^2 - 55$$

$$= 23x^2 + 5x - 62$$

Hence, the third side of the triangle is $23x^2 + 5x - 62$.

Exercise 6.2

1. Express each of the following as a monomial :

(a) $9x^2y \times 2y^2x^2 = (9 \times 2) \times (x^2y \times y^2x^2)$

$$= 18x^4y^3$$

(b) $pq \times (-p^2q^5) \times \frac{3}{2}pq = \frac{-3}{2} \times (pq \times p^2q^5 \times pq)$

$$= \frac{-3}{2}p^4q^7$$

(c) $\frac{1}{7}p^3q \times \frac{-14}{5}qr^2 \times \frac{5}{3}p^2r^3 = \left(\frac{1}{7} \times \frac{-14}{5} \times \frac{5}{3}\right) \times (p^3q \times qr^2 \times p^2r^3)$

$$= \frac{-2}{3}p^5q^2r^5$$

(d) $4xyz \times -2x^2y^2z \times 5x^3y^3z^3 = 4 \times (-2) \times 5 \times (xyz \times x^2y^2z \times x^3y^3z^3)$

$$= -40x^6y^6z^5$$

2. Solve the following expressions :

(a) $-11x(2x^3 - 3x^2 - 3x + 8)$

$$= (-11x \times 2x^3) + (-11x \times -3x^2) + (-11x \times -3x) + (-11x \times 8)$$

$$= -22x^4 + 33x^3 + 33x^2 - 88x$$

(b) $x(x - y + 1) - y(x + y - 1)$

$$= [x \times x + x \times (-y) + x \times 1] - (y \times x + y \times y + y \times (-1)]$$

$$= (x^2 - xy + x) - (xy + y^2 - y)$$

$$= x^2 - xy + x - xy - y^2 + y$$

$$= x^2 - y^2 - 2xy + x + y$$

(c) $4xy(x - y - y^2) - 2x^2y$

$$= [4xy \times x - 4xy \times y - 4xy \times y^2] - 2x^2y$$

$= 4x^2y - 4xy^2 - 4xy^3 - 2x^2y$

$= - 4xy^3 - 4xy^2 + 2x^2y$

(d) $2(y - 3) + 3(4 - 2y) - 15$

$= 2y - 6 + 12 - 6y - 15$

$= - 4y - 9$

(e) $2pq(p + q - 1) - p^2(q - 1) - pq(2 - p)$

$= 2p^2q + 2pq^2 - 2pq - p^2q + p^2 - 2pq + p^2q$

$= 2p^2q + 2pq^2 - 4pq + p^2$

3. Multiply the following binomials :

(a) $(5a + 11b)(8a + 9b)$

$= 5a \times (8a + 9b) + 11b \times (8a + 9b)$

$= 40a^2 + 45ab + 88ab + 99b^2$

$= 40a^2 + 133ab + 99b^2$

(b) $(x + 7)(x - 8)$

$= x \times (x - 8) + 7 \times (x - 8)$

$= x^2 - 8x + 7x - 56$

$= x^2 - x - 56$

(c) $(5x + 9y)(6x + 3y)$

$= 5x \times (6x + 3y) + 9y \times (6x + 3y)$

$= 30x^2 + 15xy + 54xy + 27y^2$

$= 30x^2 + 69xy + 27y^2$

(d) $(7x^3y + 5zx)(11x^3y + 8xz)$

$= 7x^3y \times (11x^3y + 8xz) + 5zx \times (11x^3y + 8xz)$

$= 77x^6y^2 + 56x^4yz + 55x^4yz + 40x^2z^2$

$= 77x^6y^2 + 111x^4yz + 40x^2z^2$

(e) $(2.5a + 0.3b)(1.5c + 0.7d)$

$= 2.5a \times (1.5c + 0.7d) + 0.3b \times (1.5c + 0.7d)$

$= 3.75ac + 1.75ad + 0.45bc + 0.21bd$

(f) $\left(\frac{2}{3}x-\frac{5}{4}y\right)\left(\frac{6}{5}x+\frac{7}{3}y\right)$

$$=\frac{2}{3}x\times\left(\frac{6}{5}x+\frac{7}{3}y\right)-\frac{5}{4}y\times\left(\frac{6}{5}x+\frac{7}{3}y\right)$$

$$=\frac{4}{5}x^2+\frac{14}{9}xy-\frac{3}{2}yx-\frac{35}{12}y^2$$

$$=\frac{4}{5}x^2+\frac{1}{18}xy-\frac{35}{12}y^2$$

4. Find the product :

(a) $(4x^2+11)(3x^2+17x-16)$

$$=4x^2\times(3x^2+17x-16)+11\times(3x^2+17x-16)$$

$$=12x^4+68x^3-64x^2+33x^2+187x-176$$

$$=12x^4+68x^3-31x^2+187x-176$$

(b) $(5pq^2-6)(6p^2q-13p+5p)$

$$=5pq^2\times(6p^2q-13p+5p)-6\times(6p^2q-13p+5p)$$

$$=30p^3q^3-65p^2q^2+25p^2q^2-36p^2q+78p-30p$$

$$=30p^3q^3-40p^2q^2-36p^2q+48p$$

(c) $(3x+4y-11)(5x-3y+8)$

$$=3x\times(5x-3y+8)+4y\times(5x-3y+8)-11\times(5x-3y+8)$$

$$=15x^2-9xy+24x+20yx-12y^2+32y-55x+33y-88$$

$$=15x^2-12y^2+11xy-31x+65y-88$$

(d) $\left(5a^2-\frac{1}{5}b^2\right)(10x+5y)$

$$=5a^2\times(10x+5y)-\frac{1}{5}b^2\times(10x+5y)$$

$$=50a^2x+25a^2y-2b^2x-b^2y$$

(e) $(x^2+y^2+z^2)(y^2-z^2)$

$$=x^2\times(y^2-z^2)+y^2\times(y^2-z^2)+z^2\times(y^2-z^2)$$

$$=x^2y^2-x^2z^2+y^4-y^2z^2+y^2z^2-z^4$$

$$=y^4-z^4+x^2y^2-x^2z^2$$

5. Simplify :

(a) $(3y + 5x - 7)(x - 1) - (y - 2x + 3)(x + 4)$

$= [3y(x - 1) + 5x(x - 1) - 7(x - 1)] - [y(x + 4) - 2x(x + 4) + 3(x + 4)]$

$= (3yx - 3y + 5x^2 - 5x - 7x + 7) - (yx + 4y - 2x^2 - 8x + 3x + 12)$

$= (3yx - 3y + 5x^2 - 12x + 7) - (yx + 4y - 2x^2 - 5x + 12)$

$= 3yx - 3y + 5x^2 - 12x + 7 - yx - 4y + 2x^2 + 5x - 12$

$= 7x^2 + 2yx - 7y - 7x + 5$

(b) $l(l + m - n) - m(l - m + n) + n(-l + m + n)$

$= l^2 + lm - ln - lm + m^2 - mn - ln + mn + n^2$

$= l^2 + m^2 + n^2 - 2ln$

(c) $(p + q)(p - q) + (p + r)(r - p) - (r^2 - q^2)$

$= p(p - q) + q(p - q) + p(r - p) + r(r - p) - (r^2 - q^2)$

$= p^2 - pq + pq - q^2 + pr - p^2 + r^2 - pr - r^2 + q^2$

$= 0$

(d) $(x + y)(y - z) + (y + z)(z - x) + (x + z)(x - y)$

$= x(y - z) + y(y - z) + y(z - x) + z(z - x) + x(x - y) + z(x - y)$

$= xy - xz + y^2 - yz + yz - xy + z^2 - xz + x^2 - xy + zx - zy$

$= x^2 + y^2 + z^2 - xy - yz - zx$

(e) $3a^2 + (3a - b)(3a + b + b^2)$

$= 3a^2 + 3a \times (3a + b + b^2) - b \times (3a + b + b^2)$

$= 3a^2 + 9a^2 + 3ab + 3ab^2 - 3ab - b^2 - b^3$

$= -b^3 + 12a^2 - b^2 + 3ab^2$

6. $2x^2(xy - 4) + 3y(x + 2)$

$= 2x^3y - 8x^2 + 3yx + 6y$

Put $x = -1$ and $y = 5$

$= 2 \times (-1)^3 \times 5 - 8 \times (-1)^2 + 3 \times 5 \times (-1) + 6 \times 5$

$= -10 - 8 - 15 + 30$

$\Rightarrow$ $-33 + 30 = -3$

7. $5a(a^3 - 8) + 2a(a + 3)$

$= 5a^4 - 40a + 2a^2 + 6a$

$= 5a^4 + 2a^2 - 36a$

Put $a = 0$

$= 5 \times (0)^4 + 2 \times (0)^2 - 36 \times 0$

$= 0 + 0 - 0 = 0$

When $a = 2$

$\therefore \quad 5 \times (2)^4 + 2 \times (2)^2 - 36 \times 2$

$= 5 \times 16 + 2 \times 4 - 72$

$= 80 + 8 - 72$

$= 88 - 72 = 16$

8. Sum of $13a + 19b + 12c$ and $14a - 21b + 11c$

$= (13a + 19b + 12c) + (14a - 21b + 11c)$

$= 27a - 2b + 23c$

Sum of $17a + 13b - 15c$ and $8a + 12b - 18c$

$= (17a + 13b - 15c) + (8a + 12b - 18c)$

$= 25a + 25b - 33c$

Now, subtract $27a - 2b + 23c$ from $25a + 25b - 33c$

$= (25a + 25b - 33c) - (25a + 25b - 33c)$

$= 0a + 0b + 0c$

$= 0 + 0 + 0 = 0$

9. Length of the cuboid $= 7a^2c$

Breadth of the cuboid $= 3a^2b^2$

Height of the cuboid $= 2abc^2$

$\therefore$ Volume of the cuboid $= l \times b \times h$

$= 7a^2c \times 3a^2b^2 \times 2abc^2$

$= 42a^5b^3c^3$ cubic unit

10. Let the required number be X.

$\therefore \quad [(x^2 - 3x)(x^3 + 4x^2 - 1)] - X = 2x^5 + x^2 - 3x$

$(x^5 + 4x^4 - x^2 - 3x^4 - 12x^3 + 3x) - X = 2x^5 + x^2 - 3x$

$(x^5 + x^4 - 12x^3 - x^2 + 3x) - X = 2x^5 + x^2 - 3x$

$(x^5 + x^4 - 12x^3 - x^2 + 3x) - (2x^5 + x^2 - 3x) = X$

$$x^5 + x^4 - 12x^3 - x^2 + 3x - 2x^5 - x^2 + 3x = X$$

$$-x^5 + x^4 - 12x^3 - 2x^2 + 6x = X$$

So, the required expression is

$$-x^5 + x^4 - 12x^3 - 2x^2 + 6x$$

Exercise 6.3

1. $(a - b)^2 = a^2 - 2ab + b^2$

$a = 9,\ b = 4$

LHS $= (a - b)^2$

$$= (9 - 4)^2 = (5)^2 = 25$$

RHS $= a^2 - 2ab + b^2$

$$= (9)^2 - 2 \times 9 \times 4 + (4)^2$$

$$= 81 - 72 + 16$$

$$= 97 - 72 = 25$$

$\therefore$ LHS = RHS **Hence verified**

2. Solve the following using a suitable identity :

(a) $(y + 8)(y + 8) = (y + 8)^2$

By using identity $(a + b)^2 = a^2 + b^2 + 2ab$

$$(y + 8)^2 = (y)^2 + (8)^2 + 2 \times y \times 8$$

$$= y^2 + 64 + 16y$$

(b) $(-2p + q)(-2p - q)$

By using identity $(a + b)(a - b) = a^2 - b^2$

$$(-2p + q)(-2p - q) = (-2p)^2 - (q)^2$$

$$= 4p^2 - q^2$$

(c) $\left(3x - \frac{1}{5}\right)\left(3x - \frac{1}{5}\right) = \left(3x - \frac{1}{5}\right)^2$

By using identity $(a - b)^2 = a^2 + b^2 - 2ab$

$$\left(3x - \frac{1}{5}\right)^2 = (3x)^2 + \left(\frac{1}{5}\right)^2 - 2 \times 3x \times \frac{1}{5}$$

$$= 9x^2 + \frac{1}{25} - \frac{6x}{5}$$

(d) $(0.5x + 0.9y)(0.5x - 0.9y)$

By using identity $(a + b)(a - b) = a^2 - b^2$

$$(0.5x + 0.9y)(0.5x - 0.9y) = (0.5x)^2 - (0.9y)^2$$

$$= 0.25x^2 - 0.81y^2$$

(e) $(2p^2 + 7a)(2p^2 - 3a)$

By using identity $(x + a)(x - b) = x^2 + (a - b)x - ab$

$$(2p^2 + 7a)(2p^2 - 3a) = (2p^2)^2 + (7a - 3a) \times 2p^2 - 7a \times 3a$$

$$= 4p^4 + 4a \times 2p^2 - 21a^2$$

$$= 4p^4 + 8ap^2 - 21a^2$$

(f) $\left(\frac{3}{2}m - \frac{2}{3}n\right)^2$

By using identity $(a - b)^2 = a^2 + b^2 - 2ab$

$$\left(\frac{3}{2}m - \frac{2}{3}n\right)^2 = \left(\frac{3}{2}m\right)^2 + \left(\frac{2}{3}n\right)^2 - 2 \times \frac{3}{2}m \times \frac{2}{3}n$$

$$= \frac{9}{4}m^2 + \frac{4}{9}n^2 - 2n$$

(g) $(p - 3q)^2$

By using identity $(a - b)^2 = a^2 + b^2 - 2ab$

$$(p - 3q)^2 = (p)^2 + (3q)^2 - 2 \times p \times 3q$$

$$= p^2 + 9q^2 - 6pq$$

(h) $(15a + 3b)^2$

By using identity $(a + b)^2 = a^2 + b^2 + 2ab$

$$(15a + 3b)^2 = (15a)^2 + (3b)^2 + 2 \times 15a \times 3b$$

$$= 225a^2 + 9b^2 + 90ab$$

3. Simplify the following :

(a) $(ax + b)^2 - (ax - b)^2$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$\therefore$ $$(ax + b)^2 - (ax - b)^2$$

$$= [(ax + b) + (ax - b)]\,[(ax + b) - (ax - b)]$$

$$= (ax + b + ax - b)(ax + b - ax + b)$$

$$= 2ax \times 2b = 4abx$$

(b) $(12a+17b)^2-(12a-17b)^2$

By using identity $a^2-b^2=(a+b)(a-b)$

$\therefore$ $(12a+17b)^2-(12a-17b)^2$

$=[(12a+17b)+(12a-17b)]\,[(12a+17b)-(12a-17b)]$

$=(12a+17b+12a-17b)\,(12a+17b-12a+17b)$

$=24a\times 34b=816ab$

(c) $(1.1m-2.1n)^2+(1.1m+2.1m)^2$

By using identity $a^2-b^2=(a+b)(a-b)$

$=[(1.1m-2.1n)+(1.1m+2.1n)]\,[(1.1m-2.1n)-(1.1m+2.1n)]$

$=[1.1m-2.1n+1.1m+2.1n]\,(1.1m-2.1n-1.1m-2.1n)$

$=2.2m\times(-4.2n)$

$=-9.24mn$

(d) $(abc+7)^2-14abc$

By using identity $(a+b)^2=a^2+b^2+2ab$

$=[(abc)^2+(7)^2+2\times abc\times 7]-14abc$

$=a^2b^2c^2+49+14abc-14abc$

$=a^2b^2c^2+49$

4. Prove that :

(a) $(4ax+3y)^2-(4ax-3y)^2=48axy$

LHS $=(4ax+3y)^2-(4ax-3y)^2$

By using identities $(a+b)^2=a^2+b^2+2ab$ and $(a-b)^2=a^2+b^2-2ab$

$=[(4ax)^2+(3y)^2+2\times 4ax\times 3y]-[(4ax)^2+(3y)^2-2\times 4ax\times 3y]$

$=(16a^2x^2+9y^2+24axy)-(16a^2x^2+9y^2-24axy)$

$=16a^2x^2+9y^2+24axy-16a^2x^2-9y^2+24axy$

$=48axy=$ RHS **Hence verified**.

(b) $(l^2+m^2)^2-4l^2m^2=(l^2-m^2)^2$

LHS $=(l^2+m^2)^2-4l^2m^2$ [$\because$ By using identity $(a+b)^2=a^2+b^2+2ab$]

$=[(l^2)^2+(m^2)^2+2l^2m^2]-4l^2m^2$

$=l^4+m^4+2l^2m^2-4l^2m^2$

$= l^4 + m^4 - 2l^2m^2$

$= (l^2 - m^2)^2$

$= \text{RHS}$ **Hence verified.**

(c) $\left(\frac{5}{x} - \frac{x}{5}\right)^2 + 2 = \frac{25}{x^2} + \frac{x^2}{25}$

LHS $\left(\frac{5}{x} - \frac{x}{5}\right)^2 + 2$

By using identity, $(a - b)^2 = a^2 + b^2 - 2ab$

$$= \left[\left(\frac{5}{x}\right)^2 + \left(\frac{x}{5}\right)^2 - 2 \times \frac{5}{x} \times \frac{x}{5}\right] + 2$$

$$= \frac{25}{x^2} + \frac{x^2}{25} - 2 + 2$$

$$= \frac{25}{x^2} + \frac{x^2}{25} = \text{RHS}$$ **Hence verified.**

(d) $(x + a)(x - a) + a^2 - x^2 = 0$

LHS $= (x + a)(x - a) + a^2 - x^2$

By using identity $(a + b)(a - b) = a^2 - b^2$

$$= [(x)^2 - (a)^2] + a^2 - x^2$$

$$= x^2 - a^2 + a^2 - x^2$$

$$= 0 = \text{RHS}$$ **Hence verified.**

5. Evaluate the following using identities :

(a) $(103)^2 = (100 + 3)^2$

By using identity $(a + b)^2 = a^2 + b^2 + 2ab$

$$(100 + 3)^2 = (100)^2 + (3)^2 + 2 \times 100 \times 3$$

$$= 10000 + 9 + 600$$

$$= 10609$$

(b) $(94)^2 = (100 - 6)^2$

By using identity $(a - b)^2 = a^2 + b^2 - 2ab$

$$(100 - 6)^2 = (100)^2 + (6)^2 - 2 \times 100 \times 6$$

$$= 10000 + 36 - 1200 = 8836$$

(c) $(10.1)^2 = (10 + 0.1)^2$

By using identity $(a + b)^2 = a^2 + b^2 + 2ab$

$$= (10)^2 + (0.1)^2 + 2 \times 10 \times 0.1$$

$$= 102.01$$

(d) $194 \times 206 = (200 - 6) \times (200 + 6)$

By using identity $(a - b)(a + b) = a^2 - b^2$

$$= (200)^2 - (6)^2$$

$$= 40000 - 36 = 39964$$

(e) $82 \times 85 = (80 + 2)(80 + 5)$

By using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(80 + 2)(80 + 5) = (80)^2 + (2 + 5) \times 80 + 2 \times 5$$

$$= 6400 + 7 \times 80 + 10$$

$$= 6400 + 560 + 10 = 6970$$

(f) $12.5 \times 10.5 = (10 + 2.5) \times (10 + 0.5)$

By using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(10 + 2.5)(10 + 0.5) = (10)^2 + (2.5 + 0.5) \times 10 + 2.5 \times 0.5$$

$$= 100 + 3.0 \times 10 + 1.25$$

$$= 100 + 30 + 1.25 = 131.25$$

(g) $(65)^2 - (25)^2$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$$(65)^2 - (25)^2 = (65 + 25)(65 - 25)$$

$$= 90 \times 40 = 3600$$

(h) $103 \times 107 = (100 + 3)(100 + 7)$

By using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(100 + 3)(100 + 7) = (100)^2 + (3 + 7) \times 100 + 3 \times 7$$

$$= 10000 + 1000 + 21 = 11021$$

6. We have,

$$x + y = 13 \text{ and } xy = 22$$

$$x^2 + y^2 = ?$$

By using identity $(x + y)^2 = x^2 + y^2 + 2xy$

Putting the given values, we get

$$(13)^2 = x^2 + y^2 + 2 \times 22$$

$$169 = x^2 + y^2 + 44$$

$$169 - 44 = x^2 + y^2$$

$$125 = x^2 + y^2$$

$$\therefore \quad x^2 + y^2 = 125$$

7. Find the continued product of :

(a) $(2x+1)(2x-1)(4x^2+1)$

By using identity, $(a+b)(a-b) = a^2 - b^2$

$$= [(2x)^2 - (1)^2](4x^2+1)$$

$$= (4x^2-1)(4x^2+1)$$

Again using same identity

$$= \left[(4x^2)^2 - (1)^2\right]$$

$$= 16x^4 - 1$$

(b) $(5+2x)(5-2x)(25-4x^2)$

By using identity, $(a+b)(a-b) = a^2 - b^2$

$$= [(5)^2 - (2x)^2](25-4x^2)$$

$$= (25-4x^2)(25-4x^2)$$

$$= (25-4x^2)^2$$

By using identity, $(a-b)^2 = a^2 + b^2 - 2ab$

$$= (25)^2 + (4x^2)^2 - 2 \times 25 \times 4x^2$$

$$= 625 + 16x^4 - 200x^2$$

8. $9x^2 - 12xy + 4y^2$

Put $x = \frac{1}{2}, y = \frac{-2}{3}$

$$\therefore \quad 9 \times \left(\frac{1}{2}\right)^2 - 12 \times \frac{1}{2} \times \left(\frac{-2}{3}\right) + 4 \times \left(\frac{-2}{3}\right)^2$$

$$= 9 \times \frac{1}{4} - 4 \times 1 \times (-1) + 4 \times \frac{4}{9}$$

$$= \frac{9}{4} + 4 + \frac{16}{9}$$

$$= \frac{81 + 144 + 64}{36} = \frac{289}{36}$$

7. Factorisation

Exercise 7.1

1. (a) $-60p - 12q = -12 \times (5p + q)$ [$\because$ HCF of 60 and 12 = 12]

$= -12(5p + q)$

(b) $19x^2 + 57x = 19x \times (x + 3)$ [$\because$ HCF of 19 and 57 = 19]

$= 19x(x + 3)$

(c) $5pq^3 - 35q^2p^2 = 5pq^2(q - 7p)$ [$\because$ HCF of 5 and 35 = 5]

(d) $36x^2y^3 - 54x^3y^2 = 18x^2y^2(2y - 3x)$ [$\because$ HCF of 36 and 54 = 18]

(e) $12a^2b^2 - 24a^2b^2 - 8a^2b = 4a^2b(3b - 6b - 2)$ [$\because$ HCF of 12, 24 and 8 = 4]

$= 4a^2b(-3b - 2)$

(f) $25x^2y - 75x^3 + 15xy^2 = 5x(5xy - 15x^2 + 3y^2)$ [$\because$ HCF of 25, 75 and 15 = 5]

(g) $9x^2 + 24x + 16x^3 = x(9x + 24 + 16x^2)$ [$\because$ HCF of 9, 24 and 16 = 1]

(h) $ab^2 + bc^2 + abc = b(ab + c^2 + ac)$

(i) $36l^2m - 63lm^2 = 9lm(4l - 7m)$

2. (a) $42(p-q)^2 + 7(p-q) = 7(p-q)[6(p-q)+1]$

$= 7(p-q)(6p - 6q + 1)$

(b) $2x^3 - 6x^2 - 36 + 12x = (2x^3 - 6x^2) + (-36 + 12x)$

$= 2x^2(x-3) + 12(-3+x)$

$= 2x^2(x-3) + 12(x-3)$

$= (x-3)(2x^2 + 12)$

$= (x-3)\, 2 \times (x^2 + 6)$

$= 2(x-3)(x^2+6)$

(c) $(x+2y)^2 - 4(2x-y)^2 = (x+2y)^2 - [2(2x-y)]^2$

By using identity $a^2 - b^2 = (a+b)(a-b)$

$= [(x+2y) + 2(2x-y)]\,[(x+2y) - 2(2x-y)]$

$= (x + 2y + 4x - 2y)\,[x + 2y - 4x + 2y]$

$= 5x\,(4y - 3x)$

(d) $14p - 18q + 36pq - 7$

$\Rightarrow \quad 14p + 36pq - 18q - 7$

$= 2p\,(7 + 18q) - 1\,(18q + 7)$

$= (18q + 7)\,(2p - 1)$

(e) $24 + a^2b + 8b + 3a^2 = 24 + 8b + 3a^2 + a^2b$

$= 8\,(3 + b) + a^2\,(3 + b)$

$= (3 + b)\,(8 + a^2)$

(f) $xy\,(a^2 + b^2) + ab\,(x^2 + y^2)$

$= xya^2 + xyb^2 + abx^2 + aby^2$

$= xya^2 + abx^2 + xyb^2 + aby^2$

$= xa\,(ay + xb) + by\,(xb + ay)$

$= (bx + ay)\,(ax + by)$

(g) $xy + yz + xz^2 + z^3$

$= y\,(x + z) + z^2\,(x + z)$

$= (x + z)\,(y + z^2)$

(h) $ab\,(p - 1) - ac\,(p - 1) = (p - 1)\,(ab - ac)$

(i) $9p^4 - 24p^2q^2 + 15q^2p^2 - 40q^4$

$= 3p^2\,(3p^2 - 8q^2) + 5q^2\,(3p^2 - 8q^2)$

$= (3p^2 - 8q^2)\,(3p^2 + 5q^2)$

Exercise 7.2

1. (a) $169 - 25y^2 = (13)^2 - (5y)^2$

By using identity $a^2 - b^2 = (a + b)\,(a - b)$

$\therefore \quad (13)^2 - (5y)^2 = (13 + 5y)\,(13 - 5y)$

(b) $64b^2 - 16b + 1 = (8b)^2 - 2 \times 8b \times 1 + (1)^2$

By using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= (8b - 1)^2 = (8b - 1)(8b - 1)$$

(c) $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$$\therefore \quad (a^2)^2 - (4b^2)^2 = (a^2 + 4b^2)(a^2 - 4b^2)$$

$$= (a^2 + 4b^2)\,[(a^2) - (2b)^2]$$

Again using same identity

$$= (a^2 + 4b^2)(a + 2b)(a - 2b)$$

(d) $9t^2 + 24pt + 16p^2 = (3t)^2 + 2 \times 3t \times 4p + (4p)^2$

By using identity $a^2 + 2ab + b^2 = (a + b)^2$

$$= (3t + 4p)^2$$

$$= (3t + 4p)(3t + 4p)$$

(e) $\frac{1}{36}x^2 - \frac{3}{5}xz + \frac{81}{25}z^2$

$$= \left(\frac{x}{6}\right)^2 - 2 \times \frac{x}{6} \times \frac{9}{5}z + \left(\frac{9}{5}z\right)^2$$

By using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= \left(\frac{x}{6} - \frac{9}{5}z\right)^2$$

$$= \left(\frac{x}{6} - \frac{9z}{5}\right)\left(\frac{x}{6} - \frac{9z}{5}\right)$$

(f) $a^2b^2 - 4b^2c^2 = b^2\,(a^2 - 4c^2)$

$$= b^2\,[(a)^2 - (2c)^2]$$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$$= b^2\,[(a + 2c)(a - 2c)]$$

(g) $81z^2 - 25y^2 = (9z)^2 - (5y)^2$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$$= (9z + 5y)(9z - 5y)$$

(h) $\frac{1}{4} + x + x^2 = \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} \times x + (x)^2$

By using identity $a^2 + 2ab + b^2 = (a + b)^2$

$$= \left(\frac{1}{2} + x\right)^2$$

$$= \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$$

(i) $4x^4 - 20x^2y^2 + 25y^4 = (2x^2)^2 - 2 \times 2x^2 \times 5y^2 + (5y^2)^2$

By using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= (2x^2 - 5y^2)^2$$

$$= (2x^2 - 5y^2)(2x^2 - 5y^2)$$

(j) $27x^3 - 75xy^2 = 3x(9x^2 - 25y^2)$

$$= 3x[(3x)^2 - (5y)^2]$$

By using identity $a^2 - b^2 = (a + b)(a - b)$

$$= 3x[(3x + 5y)(3x - 5y)]$$

(k) $4p^2 - 20pq + 25q^2 - 81r^2$

$$= [4p^2 - 20pq + 25q^2] - 81r^2$$

$$= [(2p)^2 - 2 \times 2p \times 5q + (5q)^2] - 81r^2$$

$= (2p - 5q)^2 - (9r)^2$ [Using identity $a^2 - 2ab + b^2 = (a - b)^2$]

$= [(2p - 5q) + 9r][(2p - 5q) - 9r]$ [Using identity $a^2 - b^2 = (a + b)(a - b)$]

2. (a) $x^2 + 5x + 6$

Here, $3 \times 2 = 6$ and $3 + 2 = 5$

$$= x^2 + (3 + 2)x + 6$$

$$= x^2 + 3x + 2x + 6$$

$$= x(x + 3) + 2(x + 3)$$

$$= (x + 3)(x + 2)$$

(b) $x^2 - 7x + 12$

Here, $4 \times 3 = 12$ and $4 + 3 = 7$

$$= x^2 - (4 + 3)x + 12$$

$= x^2 - 4x - 3x + 12$

$= x(x - 4) - 3(x - 4)$

$= (x - 4)(x - 3)$

(c) $x^2 - 22x - 48$

Here, $24 \times 2 = 48$ and $24 - 2 = 22$

$= x^2 - (24 - 2)x - 48$

$= x^2 - 24x + 2x - 48$

$= x(x - 24) + 2(x - 24)$

$= (x - 24)(x + 2)$

(d) $x^2 + 3xy - 18y^2$

Here, $6 \times 3 = 18$ and $6 - 3 = 3$

$= x^2 + (6 - 3)xy - 18y^2$

$= x^2 + 6xy - 3xy - 18y^2$

$= x(x + 6y) - 3y(x + 6y)$

$= (x + 6y)(x - 3y)$

(e) $7x^2 + 23xy + 6y^2$

Here, product of first and third terms

$= 7 \times 6 = 42$

So, $21 \times 2 = 42$ and $21 + 2 = 23$

$= 7x^2 + (21 + 2)xy + 6y^2$

$= 7x^2 + 21xy + 2xy + 6y^2$

$= 7x(x + 3y) + 2y(x + 3y)$

$= (x + 3y)(7x + 2y)$

(f) $x^2 - 10x + 25$

Here, $5 \times 5 = 25$ and $5 + 5 = 10$

$= x^2 - (5 + 5)x + 25$

$= x^2 - 5x - 5x + 25$

$= x(x - 5) - 5(x - 5) = (x - 5)(x - 5)$

(g) $2a^2 - 7a + 6$

Here, product of first and third terms $= 2 \times 6 = 12$

So, $4 \times 3 = 12$ and $4 + 3 = 7$

$$= 2a^2 - (4+3)\,a + 6$$

$$= 2a^2 - 4a - 3a + 6$$

$$= 2a\,(a-2) - 3\,(a-2)$$

$$= (a-2)(2a-3)$$

(h) $3x^2 + 8x - 35$

Here, product $= 3 \times 35 = 105$

So, $15 \times 7 = 105$ and $15 - 7 = 8$

$$= 3x^2 + (15-7)\,x - 35$$

$$= 3x^2 + 15x - 7x - 35$$

$$= 3x\,(x+5) - 7\,(x+5)$$

$$= (x+5)(3x-7)$$

Exercise 7.3

1. (a) $156a^4b^2c^3 \div (-4abc) = \dfrac{156a^4b^2c^3}{-4abc}$

$$= \frac{-156}{4}\,a^3bc^2$$

$$= -\,39a^3bc^2$$

(b) $-72p^3q \div 4pq = \dfrac{-72p^3q}{4pq}$

$$= \frac{-72}{4}\,p^2 = -18p^2$$

(c) $185x^3y^3z^3 \div 5xy = \dfrac{185x^3y^3z^3}{5xy}$

$$= \frac{185}{5}\,x^2y^2z^3$$

$$= 37x^2y^2z^3$$

(d) $-x^5y^9 \div -xy^4 = \dfrac{-x^5y^9}{-xy^4}$

$$= \frac{-x^4y^5}{-1} = x^4y^5$$

2. (a) $(x^5 - 7x^4 + 3x^3) \div x^2 = \dfrac{x^5 - 7x^4 + 3x^3}{x^2}$

$$= \frac{x^2\,(x^3 - 7x^2 + 3x)}{x^2}$$

$$= x^3 - 7x^2 + 3x$$

(b) $(18a^2b - 45a^3b^5) \div 9a^2b = \dfrac{18a^2b - 45a^3b^5}{9a^2b}$

$$= \frac{9a^2b\,(2 - 5ab^4)}{9a^2b}$$

$$= (2 - 5ab^4)$$

(c) $(15x^2y^2z - 12x^2yz^2) \div 4xyz = \dfrac{15x^2y^2z - 12x^2yz^2}{4xyz}$

$$= \frac{3x^2yz\,(5y - 4z)}{4xyz}$$

$$= \frac{3x}{4}\,(5y - 4z)$$

(d) $(16x^2y^2 + 12xy^2 - 8xy) \div -2xy$

$$= \frac{16x^2y^2 + 12xy^2 - 8xy}{-2xy}$$

$$= \frac{4xy\,(4xy + 3y - 2)}{-2xy}$$

$$= -2\,(4xy + 3y - 2)$$

(e) $27\,(x^2 - 5x) \div 9x\,(x - 5) = \dfrac{27\,(x^2 - 5x)}{9x\,(x - 5)}$

$$= \frac{27 \times x\,(x - 5)}{9x\,(x - 5)} = 3$$

(f) $(2x^3 - x^2) \div (2x - 1) = \dfrac{2x^3 - x^2}{2x - 1}$

$$= \frac{x^2\,(2x - 1)}{(2x - 1)} = x^2$$

3. (a) $(x^2 + 14x + 48) \div (x + 6) = \dfrac{(x^2 + 14x + 48)}{(x + 6)}$

First we solve numerator by middle spliting method.

$\therefore \qquad x^2 + 14x + 48 = x^2 + (8 + 6)\,x + 48$

$= x^2 + 8x + 6x + 48$

$= x(x + 8) + 6(x + 8)$

$= (x + 6)(x + 8)$

$\Rightarrow \quad \dfrac{(x + 6)(x + 8)}{(x + 6)} = (x + 8)$

(b) $(p^2 - 18p + 32) \div (p - 16) = \dfrac{p^2 - 18p + 32}{p - 16}$

First we solve numerator by middle spliting method.

$\therefore \quad p^2 - 18p + 32 = p^2 - (16 + 2)\, p + 32$

$= p^2 - 16p - 2p + 32$

$= p(p - 16) - 2(p - 16)$

$= (p - 16)(p - 2)$

$\Rightarrow \quad \dfrac{(p - 16)(p - 2)}{(p - 16)} = (p - 2)$

(c) $(3x^2 + 10x + 3) \div (3x + 1) = \dfrac{3x^2 + 10x + 3}{(3x + 1)}$

First we solve numerator by middle spliting method.

$\therefore \quad 3x^2 + 10x + 3 = 3x^2 + (9 + 1)\, x + 3$

$= 3x^2 + 9x + x + 3$

$= 3x(x + 3) + 1(x + 3)$

$= (x + 3)(3x + 1)$

$\Rightarrow \quad \dfrac{(x + 3)(3x + 1)}{(3x + 1)} = (x + 3)$

(d) $(x^2 - 11x + 30) \div (x - 5) = \dfrac{x^2 - 11x + 30}{x - 5}$

First we solve numerator by middle spliting method.

$\therefore \quad x^2 - 11x + 30 = x^2 - (6 + 5)\, x + 30$

$= x^2 - 6x - 5x + 30$

$= x(x - 6) - 5(x - 6)$

$= (x - 6)(x - 5)$

$\Rightarrow \quad \dfrac{(x - 6)(x - 5)}{(x - 5)} = (x - 6)$

4. (a) $x^2 + 7x + 10$ divided by $(x + 5)$

$$\Rightarrow \quad x + 5 \overline{\big)\, x^2 + 7x + 10 \,\big(}\, x + 2$$

$$\begin{array}{r} x^2 + 7x + 10 \\ \underline{x^2 + 5x} \\ _{-\quad -} \\ 2x + 10 \\ \underline{2x + 10} \\ _{-\quad -} \\ 0 \end{array}$$

$\therefore$ Quotient $= x + 2$ Remainder $= 0$

(b) $3x^2 + 10x - 8$ divided by $3x - 2$

$$\Rightarrow \quad 3x - 2 \overline{\big)\, 3x^2 + 10x - 8 \,\big(}\, x + 4$$

$$\begin{array}{r} 3x^2 + 10x - 8 \\ \underline{3x^2 - 2x} \\ _{-\quad +} \\ 12x - 8 \\ \underline{12x - 8} \\ _{-\quad +} \\ 0 \end{array}$$

$\therefore$ Quotient $= x + 4$ Remainder $= 0$

(c) $5x^3 - 4x^2 + 3x + 24$ divided by $5x + 6$

$$\Rightarrow \quad 5x + 6 \overline{\big)\, 5x^3 - 4x^2 + 3x + 24 \,\big(}\, x^2 - 2x + 3$$

$$\begin{array}{r} 5x^3 - 4x^2 + 3x + 24 \\ \underline{5x^3 + 6x^2} \\ _{-\quad -} \\ -10x^2 + 3x \\ \underline{-10x^2 - 12x} \\ _{+\quad +} \\ 15x + 24 \\ \underline{15x + 18} \\ _{-\quad -} \\ 6 \end{array}$$

$\therefore$ Quotient $= x^2 - 2x + 3$ Remainder $= 6$

(d) $4a^3 + 8a^2 + 24$ divided by $a + 4$

$$\Rightarrow \quad a + 4 \overline{\big)\, 4a^3 + 8a^2 + 24 \,\big(}\, 4a^2 - 8a + 32$$

$$\begin{array}{r} 4a^3 + 8a^2 + 24 \\ \underline{4a^3 + 16a^2} \\ _{-\quad -} \\ -8a^2 + 24 \\ \underline{-8a^2 - 32a} \\ _{+\quad +} \\ 32a + 24 \\ \underline{32a + 128} \\ _{-\quad -} \\ -102 \end{array}$$

$\therefore$ Quotient $= 4a^2 - 8a + 32$ Remainder $= -102$

8. Linear Equations in one Variable

Exercise 8.1

1. Solve the following :

(a) $17x + 12 = 13x + 24$

$$17x - 13x = 24 - 12$$
$$4x = 12$$
$$x = \frac{12}{4}$$
$$x = 3$$

(b) $0.2x + 0.3 = 0.1x + 0.5$

$$0.2x - 0.1x = 0.5 - 0.3$$
$$0.1x = 0.2$$
$$x = \frac{0.2}{0.1}$$
$$x = 2$$

(c) $7x + \frac{3}{4} = \frac{3}{2}x + 7$

$$\frac{7x}{1} - \frac{3}{2}x = \frac{7}{1} - \frac{3}{4}$$
$$\frac{14x - 3x}{2} = \frac{28 - 3}{4}$$
$$\frac{11x}{2} = \frac{25}{4}$$
$$x = \frac{25}{4} \times \frac{2}{11}$$
$$x = \frac{25}{22}$$

(d) $2a - 10 = -8$

$$2a = -8 + 10$$
$$2a = 2$$
$$a = \frac{2}{2}$$
$$a = 1$$

(e) $4(2x - 5) + 17 = 29$

$$8x - 20 + 17 = 29$$
$$8x - 3 = 29$$
$$8x = 29 + 3$$
$$8x = 32$$
$$x = \frac{32}{8}$$
$$x = 4$$

(f) $4a + 3(a + 1) = 15 - (3a - 3)$

$$4a + 3a + 3 = 15 - 3a + 3$$
$$7a + 3 = 18 - 3a$$
$$7a + 3a = 18 - 3$$
$$10a = 15$$
$$a = \frac{15}{10}$$
$$a = \frac{3}{2}$$

(g) $\left(\frac{2x - 5}{3}\right) + \left(\frac{5x - 4}{4}\right) = 1$

$$\frac{4(2x - 5) + 3(5x - 4)}{12} = 1$$
$$\frac{8x - 20 + 15x - 12}{12} = 1$$
$$\frac{23x - 32}{12} = 1$$
$$23x - 32 = 12$$
$$23x = 44$$
$$x = \frac{44}{23}$$

(h) $2m - \frac{2}{9} = 4m - \frac{4}{3}$

$$2m - 4m = \frac{-4}{3} + \frac{2}{9}$$
$$-2m = \frac{-12 + 2}{9}$$
$$-2m = \frac{-10}{9}$$
$$m = \frac{10}{9 \times 2}$$
$$m = \frac{5}{9}$$

2. Solve the following equations and verify your result :

(a) $4(2x-5)+17=13$

$$8x-20+17=13$$
$$8x-3=13$$
$$8x=13+3$$
$$8x=16$$
$$x=\frac{16}{8}$$
$$x=2$$

Check : $4(2x-5)+17=13$

$$4(2\times 2-5)+17=13$$
$$4\times(-1)+17=13$$
$$-4+17=13$$
$$13=13$$

$\therefore$ LHS = RHS **Hence verified.**

(b) $14x=9(5-6x)$

$$14x=45-54x$$
$$14x+54x=45$$
$$68x=45$$
$$x=\frac{45}{68}$$

Check : $14x=9(5-6x)$

$$14\times\frac{45}{68}=9\left(5-6\times\frac{45}{68}\right)$$
$$\frac{7\times 45}{34}=9\left(\frac{5}{1}-\frac{135}{34}\right)$$
$$\frac{315}{34}=9\left(\frac{170-135}{34}\right)$$
$$\frac{315}{34}=9\times\frac{35}{34}$$
$$\frac{315}{34}=\frac{315}{34}$$

$\therefore$ LHS = RHS **Hence verified.**

(c) $\frac{x+2}{6}+\frac{x-3}{3}=x$

$$\frac{(x+2)+2(x-3)}{6}=x$$
$$\frac{x+2+2x-6}{6}=x$$
$$\frac{3x-4}{6}=x$$
$$3x-4=6x$$
$$3x-6x=4$$
$$-3x=4$$
$$x=\frac{-4}{3}$$

Check : $\frac{x+2}{6}+\frac{x-3}{3}=x$

$$\frac{\frac{-4}{3}+2}{6}+\frac{\frac{-4}{3}-3}{3}=\frac{-4}{3}$$
$$\frac{\frac{-4+6}{3}}{6}+\frac{\frac{-4-9}{3}}{3}=\frac{-4}{3}$$
$$\frac{2}{18}+\frac{-13}{9}=\frac{-4}{3}$$
$$\frac{1}{9}-\frac{13}{9}=\frac{-4}{3}$$
$$\frac{-12}{9}=\frac{-4}{3}$$
$$-\frac{4}{3}=-\frac{4}{3}$$

$\therefore$ LHS = RHS

Hence verified.

(d)

$$\frac{3x+1}{2}+\frac{2x+5}{3}=26$$
$$\frac{3(3x+1)+2(2x+5)}{6}=26$$
$$\frac{9x+3+4x+10}{6}=26$$
$$13x+13=26\times 6$$
$$13x=156-13$$
$$13x=143$$
$$x=\frac{143}{13}$$
$$x=11$$

Check :

$$\frac{3x+1}{2}+\frac{2x+5}{3}=26$$
$$\frac{3\times 11+1}{2}+\frac{2\times 11+5}{3}=26$$
$$\frac{33+1}{2}+\frac{22+5}{3}=26$$
$$\frac{34}{2}+\frac{27}{3}=26$$
$$17+9=26$$
$$26=26$$
$\therefore$ LHS = RHS

Hence verified.

(e)

$$\frac{5x-4}{8}-\frac{x-3}{5}=\frac{x+6}{4}$$
$$\frac{5(5x-4)-8(x-3)}{40}=\frac{x+6}{4}$$
$$\frac{25x-20-8x+24}{40}=\frac{x+6}{4}$$
$$\frac{17x+4}{40}=\frac{x+6}{4}$$
$$4(17x+4)=40(x+6)$$
$$68x+16=40x+240$$
$$68x-40x=240-16$$
$$28x=224$$
$$x=\frac{224}{28}$$
$$x=8$$

Check :

$$\frac{5x-4}{8}-\frac{x-3}{5}=\frac{x+6}{4}$$
$$\frac{5\times 8-4}{8}-\frac{8-3}{5}=\frac{8+6}{4}$$
$$\frac{40-4}{8}-\frac{5}{5}=\frac{14}{4}$$
$$\frac{36}{8}-\frac{1}{1}=\frac{7}{2}$$
$$\frac{36-8}{8}=\frac{7}{2}$$
$$\frac{28}{8}=\frac{7}{2}$$
$$\frac{7}{2}=\frac{7}{2}$$
$\therefore$ LHS = RHS

Hence verified.

(f)

$$0.3(2x-3)=0.4x+1.1$$
$$0.6x-0.9=0.4x+1.1$$
$$0.6x-0.4x=1.1+0.9$$
$$0.2x=2.0$$
$$x=\frac{2}{0.2}$$
$$x=\frac{20}{2}$$
$$x=10$$

Check :

$$0.3(2\times 10-3)=0.4\times 10+1.1$$
$$0.3\times(20-3)=4.0+1.1$$
$$0.3\times 17=5.1$$
$$5.1=5.1$$
$\therefore$ LHS = RHS

Hence verified.

Exercise 8.2

1. Solve the following equations and verify the answer :

(a) $$\frac{1+x}{1-x}=\frac{5}{-4}$$

$$-4(1+x)=5(1-x)$$
$$-4-4x=5-5x$$
$$-4x+5x=5+4$$
$$x=9$$

Check : $$\frac{1+9}{1-9}=\frac{5}{-4}$$
$$\frac{10}{-8}=\frac{5}{-4}$$
$$\frac{-5}{4}=\frac{-5}{4}$$

∴ LHS = RHS **Hence verified.**

(b) $$\frac{3x+5}{2x+5}=\frac{7}{5}$$

$$5(3x+5)=7(2x+5)$$
$$15x+25=14x+35$$
$$15x-14x=35-25$$
$$x=10$$

Check : $$\frac{3\times10+5}{2\times10+5}=\frac{7}{5}$$
$$\frac{30+5}{20+5}=\frac{7}{5}$$
$$\frac{35}{25}=\frac{7}{5}$$
$$\frac{7}{5}=\frac{7}{5}$$

∴ LHS = RHS

Hence verified.

(c) $$\frac{6x+7}{3x+2}=\frac{4x+5}{2x+3}$$

$$(6x+7)(2x+3)=(4x+5)(3x+2)$$
$$12x^2+18x+14x+21=12x^2+8x+15x+10$$
$$32x+21=23x+10$$
$$32x-23x=10-21$$
$$9x=-11$$
$$x=\frac{-11}{9}$$

Check : $$\frac{6\times\left(\frac{-11}{9}\right)+7}{3\times\left(\frac{-11}{9}\right)+2}=\frac{4\times\left(\frac{-11}{9}\right)+5}{2\times\left(\frac{-11}{9}\right)+3}$$
$$\frac{\frac{-66+63}{9}}{\frac{-33+18}{9}}=\frac{\frac{-44+45}{9}}{\frac{-22+27}{9}}$$
$$\frac{-3}{-15}=\frac{1}{5}$$
$$\frac{1}{5}=\frac{1}{5}$$

∴ LHS = RHS

Hence verified.

(d) $$\frac{x-3}{x+3}=\frac{x-2}{x+2}$$

$$(x-3)(x+2)=(x-2)(x+3)$$
$$x^2+2x-3x-6=x^2+3x-2x-6$$
$$-x-6=x-6$$
$$-x-x=-6+6$$
$$-2x=0$$
$$x=0$$

Check : $$\frac{0-3}{0+3}=\frac{0-2}{0+2}$$
$$\frac{-3}{3}=\frac{-2}{2}$$
$$-1=-1$$

∴ LHS = RHS

Hence verified.

(e) $\frac{-3+x}{4x+2}=\frac{3x}{12x+1}$

$$(-3+x)(12x+1)=3x(4x+2)$$

$$-36x-3+12x^2+x=12x^2+6x$$

$$-35x-3=6x$$

$$-35x-6x=3$$

$$-41x=3$$

$$x=\frac{-3}{41}$$

Check : $\frac{-3+\left(\frac{-3}{41}\right)}{4\times\left(\frac{-3}{41}\right)+2}=\frac{3\times\left(\frac{-3}{41}\right)}{12\times\left(\frac{-3}{41}\right)+1}$

$$\frac{\frac{-123-3}{41}}{\frac{-12}{41}+2}=\frac{\frac{-9}{41}}{\frac{-36}{41}+1}$$

$$\frac{\frac{-126}{41}}{\frac{-12+82}{41}}=\frac{-\frac{9}{41}}{\frac{-36+41}{41}}$$

$$\frac{-126\times 41}{41\times 70}=\frac{-9\times 41}{41\times 5}$$

$$\frac{-126}{70}=\frac{-9}{5}$$

$$\frac{-18}{10}=\frac{-9}{5}$$

$$\frac{-9}{5}=\frac{-9}{5}$$

$\therefore$ LHS = LHS **Hence verified**.

(f) $\frac{3x+1}{2}+\frac{2x+5}{3}=26$

$$\frac{3(3x+1)+2(2x+5)}{6}=26$$

$$\frac{9x+3+4x+10}{6}=26$$

$$13x+13=26\times 6$$

$$13x=156-13$$

$$13x=143$$

$$x=\frac{143}{13}$$

$$x=11$$

Check :

$$\frac{3\times 11+1}{2}+\frac{2\times 11+5}{3}=26$$

$$\frac{33+1}{2}+\frac{22+5}{3}=26$$

$$\frac{34}{2}+\frac{27}{3}=26$$

$$17+9=26$$

$$26=26$$

$\therefore$ LHS = RHS

Hence verified.

(g) $\frac{x+5}{x}=\frac{x-7}{x-2}$

$$(x+5)(x-2)=(x-7)x$$

$$x^2-2x+5x-10=x^2-7x$$

$$3x-10=-7x$$

$$3x+7x=10$$

$$10x=10$$

$$x=\frac{10}{10}=1$$

Check : $\frac{1+5}{1}=\frac{1-7}{1-2}$

$$\frac{6}{1}=\frac{-6}{-1}$$

$$6=6$$

LHS = RHS

Hence verified.

(h) $$\frac{5x+4}{7x+3} = \frac{5x-1}{7x-2}$$

$$(5x+4)(7x-2) = (5x-1)(7x+3)$$

$$35x^2 - 10x + 28x - 8 = 35x^2 + 15x - 7x - 3$$

$$18x - 8 = 8x - 3$$

$$18x - 8x = -3 + 8$$

$$10x = 5$$

$$x = \frac{5}{10}$$

$$x = \frac{1}{2}$$

Check : $$\frac{5\times\frac{1}{2}+4}{7\times\frac{1}{2}+3} = \frac{5\times\frac{1}{2}-1}{7\times\frac{1}{2}-2}$$

$$\frac{\frac{5+8}{2}}{\frac{7+6}{2}} = \frac{\frac{5-2}{2}}{\frac{7-4}{2}}$$

$$\frac{13}{13} = \frac{3}{3}$$

$$1 = 1$$

$\therefore$ LHS = RHS **Hence verified.**

(i) $$(2x+2)(x-3) = (2x+5)(x+4)$$

$$2x^2 - 6x + 2x - 6 = 2x^2 + 8x + 5x + 20$$

$$2x^2 - 4x - 6 = 2x^2 + 13x + 20$$

$$-4x - 13x = 20 + 6$$

$$-17x = 26$$

$$x = \frac{-26}{17}$$

Check : $$\left(2\times\frac{-26}{17}+2\right)\left(\frac{-26}{17}-3\right) = \left(2\times\frac{-26}{17}+5\right)\left(\frac{-26}{17}+4\right)$$

$$\left(\frac{-52}{17}+2\right)\left(\frac{-26-51}{17}\right) = \left(\frac{-52}{17}+5\right)\left(\frac{-26+68}{17}\right)$$

$$\frac{(-52+34)}{17}\times\left(\frac{-77}{17}\right) = \left(\frac{-52+85}{17}\right)\times\left(\frac{42}{17}\right)$$

$$\frac{(-18)}{17}\times\left(\frac{-77}{17}\right) = \frac{33}{17}\times\frac{42}{17}$$

$$\frac{1386}{289} = \frac{1386}{289}$$

$\therefore$ LHS = RHS **Hence verified.**

Exercise 8.3

1. Let the first number be x.

Then second number $= 114 - x$

According to question, $114 - x = x + 14$

$$114 - 14 = x + x$$

$$100 = 2x$$

$$x = \frac{100}{2}$$

$$x = 50$$

$\therefore$ First number $= x = 50$

Second number $= 114 - x = 114 - 50 = 64$

2. Three consecutive numbers are $x, x+1, x+2$

Now, according to question,

$$8x + 8(x+1) + 8(x+2) = 96$$

$$8x + 8x + 8 + 8x + 16 = 96$$

$$24x + 24 = 96$$

$$24x = 96 - 24$$

$$24x = 72$$

$$x = \frac{72}{24}$$

$$x = 3$$

$\therefore$ Three consecutive numbers 3, $3 + 1 = 4$, $3 + 2 = 5$.

3. Let three numbers be $4x$, $5x$ and $7x$.

According to question,

Largest number + Smallest number = Third number + 72

$$7x + 4x = 5x + 72$$

$$11x = 5x + 72$$

$$11x - 5x = 72$$

$$6x = 72$$

$$x = \frac{72}{6}$$

$$x = 12$$

$\therefore$ The numbers are $4x \times 12$, 5×12 and 7×12

$\Rightarrow$ 48, 60 and 84.

4. Let the number be x.

Two-third of a number $= \frac{2}{3}x$

Three-fifth of a number $= \frac{3}{5}x$

According to question,

$$\frac{2}{3}x - \frac{3}{5}x = 6$$

$$\frac{10x - 9x}{15} = 6$$

$$\frac{1x}{15} = 6$$

$$x = 15 \times 6$$

$$x = 90$$

$\therefore$ The required number is 90.

5. Let the numerator of a rational number be x.

Denominator $= x + 5$

According to question,

$$\frac{x+11}{x+5-14} = 5$$

$$\frac{x+11}{x-9} = \frac{5}{1}$$

$$x + 11 = 5x - 45$$

$$x - 5x = -45 - 11$$

$$-4x = -56$$

$$x = \frac{56}{4}$$

$$x = 14$$

$\therefore$ Numerator $x = 14$

Denominator $x + 5 = 14 + 5 = 19$

Hence, rational number $= \frac{14}{19}$

6. Let the units digit of the number be x.

Then the tens digit of the number is $= 12 - x$

$\therefore$ Original number $= 10 \times (12 - x) + 1 \times x$

$$= 120 - 10x + x$$

$$= 120 - 9x$$

On interchanging the digits,

The new number $= 10 \times x + 1 \times (12 - x)$

$$= 10x + 12 - x$$

$$= 9x + 12$$

According to question,

$$120 - 9x + 54 = 9x + 12$$
$$120 + 54 - 12 = 9x + 9x$$
$$174 - 12 = 18x$$
$$162 = 18x$$
$$x = \frac{162}{18}$$
$$x = 9$$

$\therefore$ The units digit $= x = 9$

The tens digit$= 12 - x = 12 - 9 = 3$

Hence, the original number is 39.

7. Let Ravi's age $= x$ years

Ravi's father's age $= 3x$ years

After 12 years,

Ravi's age $= (x + 12)$ year

Ravi's father's age $= (3x + 12)$ years

According to question,

$$2(x + 12) = (3x + 12)$$
$$2x + 24 = 3x + 12$$
$$2x - 3x = 12 - 24$$
$$-x = -12$$
$$x = 12$$

$\therefore$ Ravi's age $= x = 12$ years

Ravi's father's age $= 3x = 3 \times 12 = 36$ years

8. Let Alka's age $= 3x$

Priya's age $= 2x$

Five years from now,

Alka's age $= 3x + 5$

Priya's age $= 2x + 5$

According to question,

$$\frac{3x + 5}{2x + 5} = \frac{7}{5}$$
$$5(3x + 5) = 7(2x + 5)$$
$$15x + 25 = 14x + 35$$
$$15x - 14x = 35 - 25$$
$$x = 10$$

$\therefore$ Alka's age $= 3x = 3 \times 10 = 30$ years

Priya's age $= 2x = 2 \times 10 = 20$ years

9. Let the length of the rectangular field = $4x$

and breadth = $3x$

Now, perimeter of rectangle = $2 \times (l + b)$

$$84 = 2 \times (4x + 3x)$$
$$84 = 2 \times 7x$$
$$84 = 14x$$
$$x = \frac{84}{14}$$
$$x = 6$$

$\therefore$ Length of the field = $4x = 4 \times 6 = 24$ m

Breadth of the field = $3x = 3 \times 6 = 18$ m

10. Let the total length of Nitesh's journey = x km

$$\text{Journey by train} = \frac{2}{5}x \text{ km}$$
$$\text{Journey by taxi} = \frac{1}{3}x \text{ km}$$
$$\text{Journey by bus} = \frac{1}{6}x \text{ km}$$
$$\text{Journey on foot} = 6 \text{ km}$$

Now, $$\frac{2}{5}x + \frac{1}{3}x + \frac{1}{6}x + 6 = x$$
$$\frac{2x}{5} + \frac{x}{3} + \frac{x}{6} - x = -6$$
$$\frac{12x + 10x + 5x - 30x}{30} = -6$$
$$= \frac{27x - 30x}{30}$$
$$\frac{-3x}{30} = -6$$
$$x = \frac{6 \times 30}{3}$$
$$x = 60$$

$\therefore$ The total length of his journey is 60 km.

11. Let the number of ₹ 100 notes be $2x$, number of ₹ 50 notes be $3x$ and number of ₹ 10 notes be $5x$.

$\therefore$ Total number of notes = 50

So, $$2x + 3x + 5x = 50$$
$$10x = 50$$
$$x = 5$$

$\therefore$ Amount of ₹ 100 notes = ₹ $100 \times 2x$

$$= ₹\,100 \times 2 \times 5 = ₹\,1000$$

Amount of ₹ 50 notes $= ₹\,50 \times 3x$

$= ₹\,50 \times 3 \times 5 = ₹\,750$

Amount of ₹ 10 notes $= ₹\,10 \times 5x$

$= ₹\,10 \times 5 \times 5 = ₹\,250$

$\therefore$ Total amount $= ₹(1000 + 750 + 250) = ₹\,2000$

12. Let A's share be x.

$\therefore$ B's share $= \frac{5}{6}x$

and C's share $= \frac{4}{5}$ of $\left(\frac{5}{6}x\right) = \frac{4}{5} \times \frac{5}{6}x = \frac{2}{3}x$

$$\therefore \quad \frac{x}{1} + \frac{5}{6}x + \frac{2}{3}x = 1500$$

$$\frac{6x + 5x + 4x}{6} = 1500$$

$$\frac{15x}{6} = 1500$$

$$x = \frac{1500 \times 6}{15}$$

$$x = 600$$

So, A's share $= x = ₹\,600$

B's share $= \frac{5}{6}x = \frac{5}{6} \times ₹\,600 = ₹\,500$

C's share $= \frac{2}{3}x = \frac{2}{3} \times ₹\,600 = ₹\,400$

9. Comparing Quantities

Exercise 9.1

1. (a) 25 paise : 5 rupees $= \frac{25 \text{ paise}}{5 \times 100 \text{ paise}} = \frac{25}{500} = 1:20$

(b) 450 m : 1 km 350 m $= \frac{450 \text{ m}}{1350 \text{ m}} = \frac{45}{135} = \frac{1}{3} = 1:3$

(c) 12 hours : 26 minutes $= \frac{12 \times 60 \text{ minutes}}{26 \text{ minutes}} = \frac{720}{26} = \frac{360}{13} = 360:13$

2. Sum of the ratio terms $= 7 + 8 = 15$

So, first part $= ₹\left(\frac{7}{15} \times 7500\right) = ₹\,3500$

Second part $= ₹\left(\frac{8}{15} \times 7500\right) = ₹\,4000$

3. Let the number be x,

then $16\frac{2}{3}$ of $x = 25$

or $\frac{50}{3} \times x = 25$

$$x = \frac{25 \times 3}{50}$$

$$x = \frac{3}{2}$$

Hence, the required number be $\frac{3}{2}$.

4. Total number of seats = 4200, No. of vacant seats = 1400

Number of number occupied seats = (4200 – 1400) = 2800

$\therefore$ Percentage of occupied seats $= \frac{2800}{4200} \times 100$

$$= \frac{2800}{42} = \frac{400}{6} = 66.67\%$$

5. Let Tej's original salary be ₹ x.

Percentage of increment = 10%

His new salary = ₹13750

Now, $x + x \times 10\% = 13750$

$$x + \frac{10}{100}x = 13750$$

$$\frac{100x + 10x}{100} = 13750$$

$$\frac{110x}{100} = 13750$$

$$x = \frac{13750 \times 100}{110}$$

$$x = 1250 \times 10$$

$$x = 12500$$

Hence, before increment his salary was ₹12500.

6. (a) $3 : 5 = \frac{3}{5} \times 100\% = 60\%$ (b) $5 : 12 = \frac{5}{12} \times 100\% = \frac{125}{3}\% = 41\frac{2}{3}\%$

(c) $8 : 5 = \frac{8}{5} \times 100\% = 160\%$

7. Total number of students = 35

Number of students interested in Science = 60% × 35

$$= 60\% \times 35 = \frac{60}{100} \times 35$$

$$= \frac{3}{5} \times 35 = 21 \text{ students}$$

$\therefore$ No. of students not interested in Science = 35 – 21 = 14 students

8. Original saving amount = ₹ 45000

Increased saving amount = ₹60000

Increment difference = ₹(60000 – 45000) = ₹15000

$\therefore$ Increase percentage $= \frac{15000}{45000} \times 100\% = \frac{15}{45} \times 100\%$

$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

9. Let number of boys in the school = $8x$

and number of girls in the school = $5x$

According to question,

$$8x = 168$$
$$x = \frac{168}{8}$$
$$x = 21$$

$\therefore$ No. of girls = $5x = 5 \times 21 = 105$

Hence, total strength of the school = $168 + 105 = 273$ students.

10. Let the total distance covered by him = x km.

Distance travelled by air = 5% of x

$$= 50 \times \frac{1}{100} \times x = \frac{x}{2}$$

Distance travelled by train = 35% of x

$$= \frac{35}{100} \times x = \frac{7}{20}x$$

Distance travelled by bus = 10% of x

$$= \frac{10}{100} \times x = \frac{x}{10}$$

Distance travelled by taxi $= x - \left(\frac{x}{2} + \frac{7}{20}x + \frac{x}{10}\right)$

$$= x - \left(\frac{10x + 7x + 2x}{20}\right)$$
$$= \frac{x}{1} - \frac{19x}{20} = \frac{20x - 19x}{20}$$
$$= \frac{x}{20}$$

According to question,

$$\frac{x}{20} = 144 \text{ km}$$
$$x = 144 \times 20 \text{ km}$$
$$x = 2880 \text{ km}$$

Now, distance travelled by air $= \frac{x}{2}$

$$= \frac{2880}{2} = 1440 \text{ km}$$

Distance travelled by train $= \frac{7}{20}x$

$$= \frac{7}{20} \times 2880 = 1008 \text{ km}$$

Distance travelled by bus $= \frac{x}{10}$

$$= \frac{2880}{10} = 288 \text{ km}$$

Exercise 9.2

1. Price of personal computer last year = ₹ 42000

$$\text{Decrease } \% = 15\%$$

Actual decrease = 15% of ₹ 42000

$$= ₹\left(15 \times \frac{1}{100} \times 42000\right) = ₹6300$$

∴ Price of the personal computer this year = ₹(42000 – 6300) = ₹35700

2. Let the number of mangoes the fruit seller have = x.

Number of mangoes sold = 60% of x

$$= \frac{60}{100}x = \frac{3}{5}x$$

Now,
$$x = \frac{3}{5}x + 576$$

$$x - \frac{3}{5}x = 576$$

$$\frac{5x - 3x}{5} = 576$$

$$\frac{2x}{5} = 576$$

$$x = \frac{576 \times 5}{2}$$

$$x = 1440$$

Hence, the required number of mangoes is 1440.

3. Let Yogesh's salary be ₹ x.

Then, 25% of Yogesh's salary $= \frac{25}{100} \times x = \frac{x}{4}$

∴ Deepak's salary $= x + \frac{x}{4} = \frac{4x + x}{4} = \frac{5x}{4}$

∴ Required percentage $= \frac{\frac{x}{4}}{\frac{5x}{4}} \times 100\% = \frac{x}{5x} \times 100\%$

$$= \frac{100}{5}\% = 20\%$$

4. Let Rajat's original salary be ₹ x

∴ $x + 15\% \text{ of } x = 51750$

$$x + \frac{15}{100}x = 51750$$

$$\frac{115}{100}x = 51750$$

$$x = \frac{51750 \times 100}{115}$$

$$x = 45000$$

Hence, his previous salary was ₹ 45000.

5. Total number of students = 845

Number of absent students = 169

Number of attended students = 845 – 169 = 676

$$\text{Decrease }\% = \frac{676}{845} \times 100\%$$

$$= \frac{676}{169} \times 20\% = 80\%$$

6. Present population = 180000

Increase percentage per year = 5%

Actual increase in the first year = 5% of 180000

$$= \frac{5}{100} \times 180000 = 9000$$

∴ After 1 year population = (180000 + 9000) = 189000

Now, increase in the second year = 5% of 189000

$$= \frac{5}{100} \times 189000 = 9450$$

Hence after 2 years population = (189000 + 9450) = 198450

7. Present value of the machine = ₹ 27500

Increase percentage per year = 5%

Actual increase in the first year = 5% of ₹ 27500

$$= ₹\left(\frac{5}{100} \times 27500\right) = ₹1375$$

∴ After 1 year value of the machine = ₹(27500 + 1375) = ₹ 28875

Now, increase in the second year = 5% of ₹ 28875

$$= ₹\left(\frac{5}{100} \times 28875\right) = ₹1443.75$$

Hence, after 2 years value of the machine = ₹(28875 + 1443.75) = ₹30318.75

Exercise 9.3

1. CP of the car = ₹65500

Overhead expense = ₹12500

Actual CP = ₹(65500 + 12500) = ₹78000

SP of the car = ₹84500

Here, SP > CP

∴ Profit = SP – CP

(P) = ₹(84500 – 78000) = ₹6500

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{CP}} \times 100\%$$

$$= \frac{6500}{78000} \times 100\%$$

$$= \frac{650}{78} = 8.3\%$$

2. Cost price of 8 toffees = ₹15

Cost price of 1 toffee = $₹\dfrac{15}{8}$

Selling price of 12 toffees = ₹18

Selling price of 1 toffee = $₹\dfrac{18}{12} = ₹\dfrac{3}{2}$

Here $\dfrac{15}{8} > \dfrac{3}{2}$

CP > SP

$$\text{Loss} = \text{CP} - \text{SP} = \frac{15}{8} - \frac{3}{2} = \frac{15-12}{8} = \frac{3}{8}$$

$$\therefore \text{ Loss \%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

$$= \frac{\frac{3}{8}}{\frac{15}{8}} \times 100\% = \frac{3}{15} \times 100\% = 20\%$$

3. SP of the table = ₹1500

Loss = 20%

$$\therefore \text{ CP} = \frac{\text{SP} \times 100}{100 - L\%} = \frac{1500 \times 100}{100 - 20}$$

$$= \frac{1500 \times 100}{80} = ₹1875$$

Now, in order to gain 5%, $\text{SP} = \dfrac{(100 + P\%)}{100} \times \text{CP}$

$$= \frac{(100+5)}{100} \times 1875 = \frac{105}{100} \times 1875 = ₹1968.75$$

4. CP of the furniture purchased by Shishpal = ₹45000

Overhead charges = ₹2500

Actual CP = ₹(45000 + 2500) = ₹47500

SP of the furniture sold by Shishpal = ₹50000

Here, SP > CP

Gain = (SP – CP) = ₹(5000 – 47500) = ₹2500

$$\text{Gain \%} = \frac{\text{Gain}}{\text{CP}} \times 100\% = \frac{2500}{45000} \times 100\%$$

$$= \frac{250}{45} = \frac{50}{9} = 5\frac{5}{9}\%$$

5. Let the CP of one chair = ₹x

CP of 6 chairs = ₹$6x$

SP of 8 chairs = ₹CP of 6 chairs = ₹$6x$

SP of 8 chairs = ₹$6x$

SP of 1 chair = $₹\dfrac{6x}{8} = ₹\dfrac{3}{4}x$

Here, $x > \frac{3}{4}x$

CP > SP

$$\text{Loss} = \text{CP} - \text{SP} = ₹\left(x - \frac{3}{4}x\right) = ₹\frac{x}{4}$$

$$\text{Loss } \% = \frac{\text{loss}}{\text{CP}} \times 100\% = \frac{\frac{x}{4}}{x} \times 100\% = \frac{100}{4} = 25\%$$

6. SP of the first fan = ₹1980

Gain = 10%

$$\text{SP} = \frac{110}{100}\% \text{ of CP}$$

$$\text{SP} = \frac{110}{100} \times \text{CP}$$

$\Rightarrow$ CP of the first fan $= \frac{1980 \times 100}{110} = ₹1800$

Now, SP of the second fan = ₹1980

Loss = 10%

SP = 90% of CP

$$\text{SP} = \frac{90}{100} \times \text{CP}$$

$\Rightarrow$ CP of the second fan $= \frac{100}{90} \times 1980 = \frac{10 \times 1980}{9} = ₹2200$

$\therefore$ Total CP of both the fans = ₹(1800 + 2200) = ₹4000

Total SP of both the fans = ₹2 × 1980 = ₹3960

Here, CP > SP

Loss = CP – SP = ₹(4000 – 3960) = ₹40

$$\text{Loss } \% = \frac{\text{loss}}{\text{CP}} \times 100\% = \frac{40}{4000} \times 100\% = 1\%$$

7. CP of 60 articles = ₹1200

CP of 1 article $= ₹\frac{1200}{60} = ₹20$

Profit % = 16%

SP of one article = ?

$\therefore$ $\text{SP} = \frac{(100 + P\%)}{100} \times \text{CP}$

$$= ₹\frac{(100+16)}{100} \times 20 = ₹\frac{116}{100} \times 20 = ₹\frac{116}{5} = ₹23.2$$

8. Let the CP of the bicycle sold by Abhinav be ₹x

SP of the bicycle at 7% gain $= \left(\frac{100 + P\%}{100}\right) \times \text{CP}$

$$= ₹\frac{107}{100} \times x = ₹\frac{107x}{100}$$

SP of the bicycle at 12% gain $= ₹\frac{112}{100} \times x = ₹\frac{112x}{100}$

Now, the difference between the two SPs is ₹75

$$\Rightarrow \quad \frac{112x}{100} - \frac{107x}{100} = 75$$

$$\Rightarrow \quad \frac{5x}{100} = 75$$

$$\Rightarrow \quad x = \frac{75 \times 100}{5}$$

$$x = 1500$$

Hence, the CP of the bicycle is ₹1500.

Exercise 9.4

1. Marked price of dinner set = ₹9800

Kusum paid = ₹11025

VAT = ₹(11025 – 9800) = ₹1225

$$\text{Rate of VAT} = \frac{\text{VAT}}{\text{MP}} \times 100\% = \frac{1225}{9800} \times 100\% = 12.5\%$$

2. Marked price of dress = ₹580

Discount = 20%

$$\text{Actual discount} = ₹\,580 \times 20\% = ₹\,580 \times \frac{20}{100} = ₹\,116$$

Selling price = MP – Discount = ₹(580 – 116) = ₹464

3. MP of an article = ₹600

Selling price (SP) = ₹450

Discount = MP – SP = ₹(600 – 450) = ₹150

$$\text{Rate of discount} = \frac{\text{Discount}}{\text{MP}} \times 100\% = \frac{150}{600} \times 100\% = \frac{150}{6} = 25\%$$

4. Discount = 25%

(a) MP of the table = ₹12000 = $₹\left(12000 \times \frac{25}{100}\right) = ₹3000$

∴ SP of the table = ₹(12000 – 3000) = ₹9000

(b) MP of the shoe rack = ₹8000

$$\text{Discount} = ₹8000 \times 25 \times \frac{1}{100} = ₹\,2000$$

∴ SP of the shoe rack = ₹(8000 – 2000) = ₹6000

(c) MP of the almirah = ₹6500

$$\text{Discount} = ₹6500 \times 25 \times \frac{1}{100} = ₹1625$$

∴ SP of the almirah = ₹(6500 – 1625) = ₹4875

5. CP of washing machine = ₹ 14400

$$\text{Profit } \% = 10\%$$

$$\text{SP of washing machine} = \left(\frac{100 + P\%}{100}\right) \times \text{CP}$$

$$= ₹\left(\frac{100 + 10}{100}\right) \times 14400$$

$$= ₹\frac{110}{100} \times 14400 = ₹15840$$

Let the marked price be x

$$\text{Discount } \% = 12\,\%$$

$$\text{Actual discount} = 12\% \text{ of } x = \frac{12x}{100}$$

$$\text{Selling price} = \text{MP} - \text{Discount}$$

$$= x - \frac{12x}{100} = \frac{88x}{100}$$

But we have SP = ₹ 15840

$$\therefore \quad \frac{88x}{100} = 15840$$

$$x = \frac{15840 \times 100}{88}$$

$$x = ₹18000$$

Hence, the marked price of the washing machine should be ₹ 18000.

6. SP of the computer = ₹ 25920

Let the MP be ₹ x

$$\text{VAT} = 8\% \text{ of } x = \frac{8x}{100}$$

$$\text{SP} = \text{MP} + \text{VAT}$$

$$= x + \frac{8x}{100} = \frac{108x}{100}$$

But, SP = ₹ 25920

$$\therefore \quad \frac{108x}{100} = 25920$$

$$x = \frac{25920 \times 100}{108}$$

$$x = ₹24000$$

Hence, the MP of the computer is ₹ 24000.

7. MP of the crockery = ₹ 6500

$$\text{Discount} = 8\%$$

$$\text{Actual discount} = ₹6500 \times \frac{8}{100} = ₹520$$

$$\therefore \quad \text{SP of the crockery} = \text{MP} - \text{Discount}$$

$$= ₹(6500 - 520) = ₹5980$$

Now, sale tax = 8%

$$\text{Actual tax} = ₹\left(8 \times \frac{1}{100} \times 5980\right) = 478.4$$

$\therefore$ Paid amount = ₹(5980 + 478.4) = ₹6458.4

8. (a) Cost of sports shoes = ₹850

$$\text{VAT} = 8\% \text{ of } ₹850$$

$$= ₹\left(\frac{8}{100} \times 850\right) = ₹68$$

SP of shoes = ₹(850 + 68) = ₹918

(b) Cost of leather jacket = ₹2550

$$\text{VAT} = 10\% \text{ of } ₹2550$$

$$= ₹\left(\frac{10}{100} \times 2550\right) = ₹255$$

SP of leather jacket = ₹(2550 + 255) = ₹2805

(c) Cost of cotton clothes = ₹800

$$\text{VAT} = 5\% \text{ of } ₹800$$

$$= ₹\left(\frac{5}{100} \times 800\right) = ₹40$$

SP of cotton clothes = ₹(800 + 40) = ₹840

So, total amount of the bill = ₹(918 + 2805 + 840) = ₹4563

Exercise 9.5

1. Principal (P) = ₹4000; Time (n) = 3 years; Rate (R) = 10%

$$\therefore \quad \text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

$$= ₹4000\left(1 + \frac{10}{100}\right)^3 = ₹4000\left(1 + \frac{1}{10}\right)^3$$

$$= ₹4000 \times \left(\frac{11}{10}\right)^3 = ₹4000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

$$= ₹4 \times 11 \times 11 \times 11 = ₹5324$$

$$\therefore \quad CI = A - P$$

$$= ₹(5324 - 4000) = ₹1324$$

2. $P = ₹100$; $R = 10\%$; $n = 1\frac{1}{2}$ years $= \frac{3}{2}$ years

If interest is compounded half yearly, then

$$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

$$= ₹100\left(1+\frac{10}{200}\right)^{2\times\frac{3}{2}}$$

$$= ₹100\left(1+\frac{1}{20}\right)^{3}$$

$$= ₹100\times\left(\frac{21}{20}\right)^{3}$$

$$= ₹100\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

$$= ₹\frac{21\times 21\times 21}{80} = ₹115.7625$$

3. $P = ₹3600$

$n = 3$ years

$R = 25\%$

$\therefore$ $$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$= ₹3600\left(1+\frac{25}{100}\right)^{3}$$

$$= ₹3600\times\left(\frac{125}{100}\right)^{3}$$

$$= ₹3600\times\frac{125}{100}\times\frac{125}{100}\times\frac{125}{100}$$

$$= ₹\frac{70312500}{100\times 100} = ₹7031.25$$

4. $P = ₹25000$

$R = 15\%$

$n = 2$ years 6 months $= 2\frac{1}{2}$ years

For 2 years

$\therefore$ $$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$= ₹25000\left(1+\frac{15}{100}\right)^{2}$$

$$= ₹25000\times\left(\frac{115}{100}\right)^{2}$$

$$= ₹25000\times\frac{115}{100}\times\frac{115}{100}$$

$$= ₹\frac{25\times 115\times 115}{10} = ₹33062.5$$

Now, for $\frac{1}{2}$ year,

$$P = ₹33062.5$$

$$R = 15\%, T = \frac{1}{2} \text{ year}$$

$$SI = \frac{P \times R \times T}{100}$$

$$= ₹\frac{33062.5 \times 15 \times 1}{100 \times 2}$$

$$= ₹\frac{495937.5}{200} = ₹2479.68$$

$\therefore$ Amount for $2\frac{1}{2}$ years

$$A = ₹(33062.5 + 2479.68) = ₹35542.18$$

Hence, $\quad CI = A - P$

$$= ₹(35542.18 - 25000) = ₹10542.18$$

5. $P = ₹30000$

$R = 15\%$

$T = 2$ years 4 months $= 2$ years and $\frac{1}{3}$ year

For 2 years $\quad A = P\left(1 + \frac{R}{100}\right)^n$

$$= ₹30000\left(1 + \frac{15}{100}\right)^2$$

$$= ₹30000 \times \left(\frac{115}{100}\right)^2$$

$$= ₹30000 \times \frac{115}{100} \times \frac{115}{100}$$

$$= ₹3 \times 115 \times 115 = ₹39675$$

For $\frac{1}{3}$ year, $\quad P = ₹39675$

$$R = 15\%, \quad T = \frac{1}{3} \text{ year}$$

$$SI = \frac{P \times R \times T}{100}$$

$$= ₹\frac{39675 \times 15 \times 1}{100 \times 3}$$

$$= ₹\frac{198375}{100} = ₹1983.75$$

$\therefore$ Amount for 2 years and 4 months

$$A = ₹(39675 + 1983.75) = ₹41658.75$$

Hence, $\quad CI = A - P$

$$= ₹(41658.75 - 30000) = ₹11658.75$$

6. $P = ₹60000$

$R = 8\%$

$n = 1$ year

(a) If the interest is compounded half yearly, then

$$A = P\left(1+\frac{8}{200}\right)^{2n}$$

$$= ₹60000\left(1+\frac{R}{200}\right)^{2}$$

$$= ₹60000 \times \frac{208}{200} \times \frac{208}{200} = ₹64896$$

(b) If the interest is compounded quarterly, then

$$A = P\left(1+\frac{R}{400}\right)^{4n}$$

$$= ₹60000\left(1+\frac{8}{400}\right)^{4\times 1}$$

$$= ₹60000\left(1+\frac{1}{50}\right)^{4}$$

$$= ₹60000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}$$

$$= ₹\frac{40591206}{625} = ₹64945.92$$

$\therefore$ differene between the amounts $= ₹(64945.92 - 64896)$

$= ₹49.92$

7. $P = ₹4000$

$A = ₹5290$

$n = 2$ years

$R = ?$

$\therefore$
$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$5290 = 4000\left(1+\frac{R}{100}\right)^{2}$$

$$\frac{5290}{4000} = \left(1+\frac{R}{100}\right)^{2}$$

$$\left(\frac{23}{20}\right)^{2} = \left(1+\frac{R}{100}\right)^{2}$$

Taking square root, we get

$$1+\frac{R}{100} = \frac{23}{20}$$

$$\frac{R}{100} = \frac{23}{20} - 1$$

$$\frac{R}{100} = \frac{23-20}{20}$$

$$\frac{R}{100} = \frac{3}{20} \times 100$$

$$R = 15\%$$

8. $P = ?$

$A = ₹37044$

$R = 10\%$

$n = 1\frac{1}{2}$ years $= \frac{3}{2}$ years

For half yearly,

$$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

$$37044 = P\left(1 + \frac{10}{200}\right)^{2\times\frac{3}{2}}$$

$$37044 = P\left(1 + \frac{1}{20}\right)^{3}$$

$$37044 = P \times \left(\frac{21}{20}\right)^{3}$$

$$37044 \times \left(\frac{20}{21}\right)^{3} = P$$

$$37044 \times \frac{20 \times 20 \times 20}{21 \times 21 \times 21} = P$$

$$4 \times 20 \times 20 \times 20 = P$$

$$P = ₹32000$$

9. $P = ₹8000$

$A = ₹9261$

$R = 5\%$

$n = ?$

$$A = P\left(1 + \frac{R}{100}\right)^{n}$$

$$9261 = 8000\left(1 + \frac{5}{100}\right)^{n}$$

$$\frac{9261}{8000} = \left(1 + \frac{1}{20}\right)^{n}$$

$$\left(\frac{21}{20}\right)^{3} = \left(\frac{21}{20}\right)^{n}$$

Equating the powers

$$n = 3 \text{ years}$$

10. Value of the car $(P) = ₹150000$

$$R = 5\%$$

$$n = 3 \text{ years}$$

$$A = P\left(1 - \frac{R}{100}\right)^n \qquad [\because \text{ depreciation}]$$

$$= ₹150000\left(1 - \frac{5}{100}\right)^3$$

$$= ₹150000 \times \left(\frac{95}{100}\right)^3$$

$$= ₹150000 \times \frac{95}{100} \times \frac{95}{100} \times \frac{95}{100}$$

$$= ₹\frac{12860625}{100} = ₹128606.25$$

Hence, the depreciated value of the car in the year 2012 is ₹128606.25

11. $A = 48400;\ P = 40000;\ R = 10\%;\ n = ?$

$$A = P\left(1 + \frac{R}{100}\right)^n \qquad [\because \text{ Appreciation}]$$

$$48400 = 40000\left(1 + \frac{10}{100}\right)^n$$

$$\frac{48400}{40000} = \left(1 + \frac{1}{10}\right)^n$$

$$\frac{484}{400} = \left(\frac{11}{10}\right)^n$$

$$\left(\frac{121}{100}\right) = \left(\frac{11}{10}\right)^n$$

$$\left(\frac{11}{10}\right)^2 = \left(\frac{11}{10}\right)^n$$

Equating the powers, $n = 2$ years

12. $P = ₹75000$

$R = 10\%$

$n = 3$ years

$$\therefore \qquad A = P\left(1 - \frac{R}{100}\right)^n \qquad [\because \text{ Depreciation}]$$

$$= ₹75000\left(1 - \frac{10}{100}\right)^3$$

$$= ₹75000\left(\frac{9}{10}\right)^3$$

$$= ₹75000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = ₹54675$$

10. Direct and Indirect Variation

Exercise 10.1

1.

x	7	x_2	21	1.5
y	21	57	y_3	y_4

Since the variation is direct, we have

$$\frac{7}{21} = \frac{x_2}{57}$$

$$\Rightarrow \quad x_2 = 7 \times \frac{57}{21}$$

$$x_2 = 19$$

and $$\frac{7}{21} = \frac{21}{y_3}$$

$$y_3 = \frac{21 \times 21}{7}$$

$$y_3 = 63$$

Also, $$\frac{7}{21} = \frac{1.5}{y_4}$$

$$\Rightarrow \quad y_4 = \frac{21 \times 1.5}{7}$$

$$y_4 = 4.5$$

2. Since the variation between notebooks and cost is given to be direct.

Number of notebooks	17	13
Cost (in ₹)	212.50	y_2

Since, the variation is direct so we have

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{17}{212.50} = \frac{13}{y_2}$$

$$\Rightarrow \quad y_2 = \frac{212.50 \times 13}{17} = ₹162.5$$

Hence, the cost of 13 notebooks is ₹ 162.5.

3. If the weight increases, the number of toys will also increase. So it is a direct variation.

Number of toys	4	x_2
Weight (in g)	600	1800

Since, the variation is direct, we have

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{4}{600} = \frac{x_2}{1800}$$

$\Rightarrow \quad x_2 = 4 \times \frac{1800}{600}$

$x_2 = 12 \text{ toys}$

Hence, 12 toys would weigh 1800 g.

4. If the time increases, then number of containers will also increase. So it is a direct variation.

Number of containers	18	x_2
Time (in hours)	6	48

Since, the variation is direct, we have

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{18}{6} = \frac{x_2}{48}$$

$$\Rightarrow \quad x_2 = \frac{48 \times 18}{6}$$

$$x_2 = 144 \text{ containers}$$

Hence, 144 containers can be filled in 48 hours.

5. Since the variation distance and time is given to be direct.

Distance (in m)	140	560
Time (in minutes)	60	y_2

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{140}{60} = \frac{560}{y_2}$$

$$\Rightarrow \quad y_2 = \frac{60 \times 560}{140}$$

$$y_2 = 240 \text{ minutes}$$

Hence, Karan will cover 560 m distance in 240 minutes.

6. Since the variation is direct between quantity of flour and persons.

Quantity of flour (in kg)	120	x_2
Number of persons	6	18

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{120}{6} = \frac{x_2}{18}$$

$$\Rightarrow \quad x_2 = \frac{120 \times 18}{6}$$

$$x_2 = 360 \text{ kg}$$

Hence, 360 kg flour will be required for 18 persons.

7. The distance travelled is more if the quantity of petrol used is more. So, the variation is direct.

Petrol (in L)	15	22	x
Distance (in km)	255	y	442

(a)
$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$
$$\frac{15}{255} = \frac{22}{y}$$
$$\Rightarrow \quad y = \frac{255 \times 22}{15}$$
$$y = 374 \text{ km}$$

(b)
$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$
$$\frac{15}{255} = \frac{x}{442}$$
$$\Rightarrow \quad x = \frac{15 \times 442}{255}$$
$$x = 26\, l$$

8. If the charge increases then distance will also increase. So the variation is direct.

Charges (in ₹)	375	562.5
Distance (in km)	150	y_2

Since, the variation is direct, we have
$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$
$$\frac{375}{150} = \frac{562.5}{y_2}$$
$$\Rightarrow \quad y_2 = \frac{150 \times 562.5}{375}$$
$$y_2 = 225 \text{ km}$$

Hence, 225 km distance can be travelled for ₹ 562.5 by the taxi.

9. Since the mass of rod varies directly to its length, so it is a direct variation.

Length (in cm)	15	x_2
Mass (in g)	180	105

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$$
$$\frac{15}{180} = \frac{x_2}{105}$$
$$\Rightarrow \quad x_2 = \frac{15 \times 105}{180}$$
$$x_2 = 8.75 \text{ cm}$$

Hence, the length of a rod of mass 105 g is 8.75 cm.

10. The distance on the map and actual distance have a direct variation.

Distance on the map (in cm)	0.6	70.5
Actual distance (in km)	6.6	y_2

$$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{0.6}{6.6} = \frac{70.5}{y_2}$$

$$\Rightarrow \quad y_2 = \frac{6.6 \times 70.5}{0.6}$$

$$y_2 = 775.5 \text{ km}$$

Hence, the actual distance between these points is 775.5 km.

Exercise 10.2

1.

x	18	36	24	x_4	x_5
y	40	y_2	y_3	16	60

If x and y are in inverse variation, then

(i)
$$x_1 y_1 = x_2 y_2$$
$$18 \times 40 = 36 \times y_2$$
$$y_2 = \frac{18 \times 40}{36}$$
$$y_2 = 20$$

(ii)
$$x_1 y_1 = x_3 y_3$$
$$18 \times 40 = 24 \times y_3$$
$$y_3 = \frac{18 \times 40}{24}$$
$$y_3 = 30$$

(iii)
$$x_1 y_1 = x_4 y_4$$
$$18 \times 40 = x_4 \times 16$$
$$x_4 = \frac{18 \times 40}{16}$$
$$x_4 = 45$$

(iv)
$$x_1 y_1 = x_5 y_5$$
$$18 \times 40 = x_5 \times 60$$
$$x_5 = \frac{18 \times 40}{60}$$
$$x_5 = 12$$

2. If the number of men increases, the number of days will decrease and it is a case of inverse variation.

Number of men (x)	4	$4 + 6 = 10$
Number of days (y)	10	y_2

Since the variation is inverse, we have

$$x_1 y_1 = x_2 y_2$$
$$4 \times 10 = 10 \times y_2$$
$$y_2 = \frac{4 \times 10}{10}$$
$$y_2 = 4 \text{ days}$$

Hence, they will complete the job in 4 days.

3. When the number of cows increases, the number of days will decrease. Hence, the variation is inverse.

Number of cows (x)	30	x_2
Number of days (y)	21	15

$$\therefore \quad x_1y_1 = x_2y_2$$

$$30 \times 21 = x_2 \times 15$$

$$x_2 = \frac{30 \times 21}{15}$$

$$x_2 = 42 \text{ cows}$$

Hence, 42 cows will graze the same field in 15 days.

4. When number of students decreases, the number of days will increase. Hence the variation is inverse.

Number of students (x)	210	$210 - 60 = 150$
Number of days (y)	60	y_2

$$\therefore \quad x_1y_1 - 210 \times 10 = x_2y_2 \qquad [\because \text{After 10 days food consumed} = 210 \times 10 = 2100]$$

$$210 \times 60 - 2100 = 150 \times y_2$$

$$12600 - 2100 = 150 \times y_2$$

$$10500 = 150y_2$$

$$y_2 = \frac{10500}{150} = 70 \text{ days}$$

Hence, the remaining provisions will last for 70 days.

5. If the number of men increases, the number of days will decrease. So the variation is inverse.

Number of men (x)	39	45
Number of days (y)	15	y_2

$$\therefore \quad x_1y_1 = x_2y_2$$

$$39 \times 15 = 45 \times y_2$$

$$y_2 = \frac{39 \times 15}{45}$$

$$y_2 = 13 \text{ days}$$

But if the length of river is doubled then required number of days = $13 \times 2 = 26$ days.

6. If the speed increases, the time taken will decrease. This is a case of inverse variation.

Speed of car in (km/h) (x)	60	80
Time taken (in hours) (y)	2	y_2

$$\therefore \quad x_1y_1 = x_2y_2$$

$$60 \times 2 = 80 \times y_2$$

$$y_2 = \frac{60 \times 2}{80} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2} \text{ hours}$$

Hence, the car will take $1\frac{1}{2}$ hours to reach the destination.

7. If the number of men decreases, the number of days will increase. So the variation is inverse.

Number of men (x)	18	16
Number of days (y)	20	y_2

$$\therefore \quad x_1 y_1 = x_2 y_2$$

$$18 \times 20 = 6 \times y_2$$

$$y_2 = \frac{18 \times 20}{6} = 60$$

Hence, 15 men will take 60 days to reap the field.

8. If the cost increases, the number of suitcases will decrease. So the variation is inverse.

Number of suitcases (x)	42	x_2
Cost (in ₹) (y)	500	600

$$\therefore \quad x_1 y_1 = x_2 y_2$$

$$42 \times 500 = 600 \times x_2$$

$$x_2 = \frac{42 \times 500}{600} = \frac{42 \times 5}{6} = 35 \text{ suitcases}$$

Hence, he would be able to buy 35 suitcases now.

Exercise 10.3

1. In a day Ankit will complete the work $= \frac{1}{16}$ part

In a day Seta will complete the work $= \frac{1}{18}$ part

In a day Ravi will complete the work $= \frac{1}{24}$ part

They will complete the work together $= \frac{1}{16} + \frac{1}{18} + \frac{1}{24}$

$$= \frac{9+8+6}{144} = \frac{23}{144} \text{ part}$$

$\therefore$ Required number of days $= \frac{144}{23}$ days

$$= 6\frac{6}{23} \text{ days}$$

2. Part of tank filled by tap A in 1 hour $= \frac{1}{12}$

Part of tank filled by tap B in 1 hour $= \frac{1}{6}$

Both taps A and B filled it in 1 hour $= \frac{1}{12} + \frac{1}{6} = \frac{1+2}{12} = \frac{3}{12} = \frac{1}{4}$ part

Hence, time taken by both the taps to fill the tank together is 4 hours.

3. Work done by Ram in 1 day $= \frac{1}{15}$

Work done by Ashish in 1 day $= \frac{1}{12}$

Both of them work in 1 day $= \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60} = \frac{3}{20}$

Now, work done by both of them in 4 days $= 4 \times \frac{3}{20} = \frac{3}{5}$

Work left $= 1 - \frac{3}{5} = \frac{2}{5}$

This work is finished by Ram in some number of days.

Time taken by Ram to do 1 work = 15 days

Time taken by Ram to do $\frac{2}{5}$ of work $= \frac{2}{5} \times 15 = 6$ days

Hence, Ram will complete the remaining work in 6 days.

4. Work done by Anurag in 1 day $= \frac{1}{30}$

Work done by Arushi in 1 day $= \frac{1}{25}$

Both of them work in 1 day $= \frac{1}{30} + \frac{1}{25} = \frac{5+6}{150} = \frac{11}{150}$

Now, work done by both of them in 10 days $= 10 \times \frac{11}{150} = \frac{11}{15}$

Work left $= 1 - \frac{11}{15} = \frac{4}{15}$

$\therefore$ Time taken by Arushi to do 1 work = 25 days

$\therefore$ Time taken by Arushi to do $\frac{4}{15}$ work $= \frac{4}{15} \times 25 = \frac{20}{3}$ days

$= 6\frac{2}{3}$ days

Hence, Arushi will do the remaining work in $6\frac{2}{3}$ days.

5. Speed of bus = 45 km/h

$= 45 \times \frac{1000}{60 \times 60} \frac{\text{m}}{\text{sec}}$

$= 45 \times \frac{5}{18}$ m/s

Time = 36 seconds

$\therefore$ Distance = Speed $\times$ Time

$= 45 \times \frac{5}{18} \times 36$ m

$= 45 \times 10$ m $= 450$ m

Hence, the bus will cover 450 m distance in 36 seconds.

6. Distance = 500 m

Speed of train = 72 km/h

$$= 72 \times \frac{5}{18} \text{ m/sec} = 20 \text{ m/sec}$$

$\therefore$ $\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{500}{20}$ seconds = 25 seconds

Hence, the train will take 25 seconds to cross the tree.

7. Distance = Sum of lengths of two trains

$= (180 + 360)$ m = 540 m

$\therefore$ Speed = (49 – 34) km/h

= 15 km/h

$= 15 \times \frac{5}{18} \text{ m/sec} = \frac{25}{6} \text{ m/sec}$

Time taken by the first train to overtake the second train $= 540 \div \frac{25}{6}$ $\left[\because t = \frac{d}{5}\right]$

$= 540 \times \frac{6}{25} = \frac{648}{5} = 129\frac{3}{5}$ seconds

$=$ (or) 2 min $9\frac{3}{5}$ seconds

Hence, the required time is 2 min $9\frac{3}{5}$ seconds.

11. Understanding Quadrilaterals

Exercise 11.1

1. (a) Concave polygon (b) Convex polygon
(c) Simple closed curve (d) Open curve
(e) Not a polygon because its sides intersect each other at points other than the end points.
(f) Not a polygon because one of its parts is made of curved lines.
(g) Convex polygon (closed curve)

2. (a) 5 sides $n = 5$

Sum of interior angles of n sided Polygon

$= (2n - 4)$ right angle

$= (2n - 4) \times 90^\circ$

$= (2 \times 5 - 4) \times 90^\circ$

$= 6 \times 90^\circ = 540^\circ$

(b) 10 sides $n = 10$

Sum of interior angles of n sided polygon $= (2n - 4) \times 90^\circ$

$= (2 \times 10 - 4) \times 90^\circ$

$= 16 \times 90^\circ = 1440^\circ$

(c) 16 sides $n = 16$

Sum of interior angles of n sided polygen $= (2n - 4) \times 90^\circ$

$= (2 \times 16 - 4) \times 90^\circ$

$= 28 \times 90^\circ = 2520^\circ$

3. (a) 144°

Interior angle $= 144^\circ$

Exterior angle $= 180^\circ - 144^\circ = 36^\circ$

$\therefore$ Number of sides $= \dfrac{360^\circ}{\text{Exterior angle}}$

$= \dfrac{360^\circ}{36^\circ} = 10$ sides

(b) 160°

Interior angle $= 160^\circ$

Exterior angle $= 180 - 160 = 20^\circ$

$\therefore$ Number of sides $= \dfrac{360^\circ}{\text{Exterior angle}}$

$= \dfrac{360^\circ}{20^\circ} = 18$ sides

(c) 156°

Interior angle $= 156^\circ$

Exterior angle $= 180 - 156 = 24^\circ$

$\therefore$ Number of sides $= \dfrac{360^\circ}{\text{Exterior angle}}$

$= \dfrac{360^\circ}{24^\circ} = 15$ sides

4. Let the angles of pentagon $1x, 3x, 5x, 4x$ and $5x$.

$\therefore$ Sum of the interior angles of a pentagon $= 540^\circ$

$$1x + 3x + 5x + 4x + 5x = 540^\circ$$

$$18x = 540^\circ$$

$$x = \frac{540^\circ}{18}$$

$$x = 30^\circ$$

So, the angles are :

$1 \times 30^\circ, 3 \times 30^\circ, 5 \times 30^\circ, 4 \times 30^\circ$ and $5 \times 30^\circ$

$30^\circ, 90^\circ, 150^\circ, 120^\circ$ and 150°

5. Let the angles be x° each.

$\therefore$ Sum of adjacent angles of a parallelogram $= 180^\circ$

$$\angle A + \angle B = 180^\circ$$

$$x + x = 180^\circ$$

$$2x = 180^\circ$$

$$x = \frac{180^\circ}{2}$$

$$x = 90^\circ$$

$\therefore \quad \angle A = \angle C = 90^\circ$ [Opposite angles are equal]

$\angle B = \angle D = 90^\circ$

6. Exterior angle = 25°

$\therefore$ Exterior angle $= \dfrac{360^\circ}{\text{Number of sides}}$

$$25^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{25}$$

$$n = 14.4$$

As $n = 14.4$ is not a natural number, so 25° is not the exterior angle of a regular polygon.

7. 60° is the minimum interior angle possible for a regular polygon.

8. 120° is the maximum exterior angle possible for a regular polygon.

9. (a) Sum of angles of quadrilaterals = 360°

$$x + 82^\circ + 73^\circ + 97^\circ = 360^\circ$$

$$x + 252^\circ = 360^\circ$$

$$x = 360^\circ - 252^\circ$$

$$x = 108^\circ$$

(b) Sum of angles of pentagon = 540°

$$3x + 2x + 3x + 4x + 6x = 540^\circ$$

$$18x = 540^\circ$$

$$x = \frac{540^\circ}{18}$$

$$x = 30^\circ$$

(c) $$x + 60 + 90 + (180 - 110) = 360^\circ$$

$$x + 60^\circ + 90^\circ + 70^\circ = 360^\circ$$

$$x + 220^\circ = 360^\circ$$

$$x = 360^\circ - 220^\circ$$

$$x = 140^\circ$$

(d) $$x + x + x + x + x + x = 720^\circ$$

$$6x = 720^\circ$$

$$x = \frac{720^\circ}{6}$$

$$x = 120^\circ$$

10. (a) Angle $x = 180^\circ - 50^\circ$ (linear pair)

$= 130^\circ$

Angle $y = 180^\circ - 50^\circ$ (linear pair)

$= 130^\circ$

Angle $z = 180^\circ - 80^\circ$ (linear pair)

$= 100^\circ$

$\therefore \quad x + y + z = 130 + 130 + 100 = 360^\circ$

(b) Angle $x = 180° - 70° = 110°$ (Linear pair)

Angle $z = 180° - 60° = 120°$ (Linear pair)

Angle $y = (60° + 70°) = 130°$ [Sum of interior angles]

$\therefore$ $x + y + z = 110° + 120° + 130° = 360°$

(c) $x = 30° + 90° = 120°$ [Sum of interior angles]

$y = 180° - 30° = 150°$ (Linear pair)

$z = 180° - 90° = 90°$ (Linear pair)

$\therefore$ $x + y + z = 120° + 150° + 90° = 360°$

11. (a) Sum of interior angles of quadrilateral $= 360°$

$\angle BAD + 110° + 80° + 70° = 360°$

$\angle BAD + 260° = 360°$

$\angle BAD = 360° - 260° = 100°$

$\therefore$ $x = 180 - 110 = 70°$ (Linear pair)

$y = 180° - 100° = 80°$ (Linear pair)

$z = 180° - 70° = 110°$ (Linear pair)

$w = 180° - 80° = 100°$ (Linear pair)

(b) Sum of interior angles of quadrilateral $= 360°$

$x = 180 - 70 = 110°$ (Linear pair)

$y = 180 - 95 = 85°$ (Linear pair)

$z = 180 - 110 = 70°$ (Linear pair)

Now, sum of interior angles of quadrilateral $= 360°$

$70° + 95° + 110° + \angle PSR = 360°$

$\angle PSR = 360° - 275 = 85°$

$\therefore$ $w = 180° - 85° = 95°$ (Linear pair)

Exercise 11.2

1. Let the angles be $1x, 3x, 7x$ and $9x$.

$\therefore$ Sum of the angles of quadrilateral $= 360°$

$$1x + 3x + 7x + 9x = 360°$$

$$20x = 360°$$

$$x = \frac{360°}{20}$$

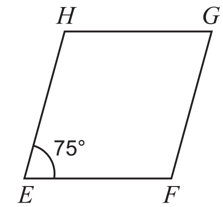
$$x = 18°$$

$\therefore$ Angles are

$1 \times 18°, 3 \times 18°, 7 \times 18°$ and $9 \times 18°$

$18°, 54°, 126°$ and $162°$.

2. $\therefore \quad \angle E = 75^\circ$



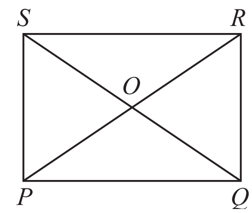
Sum of adjacent angles in $||^{gm} = 180^\circ$

$$\angle E + \angle F = 180^\circ$$
$$75^\circ + \angle F = 180^\circ$$
$$\angle F = 180^\circ - 75^\circ = 105^\circ$$

So, $\angle E = \angle G = 75^\circ$ [Opposite angles of || gm]

$\angle F = \angle H = 105^\circ$

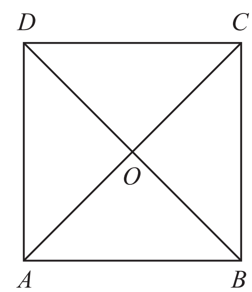
3. $\because \quad PQ = (5x - 6)$



Since, diagonals of rectangle bisect each other

$$PO = OQ$$
$$5x - 6 = 3x + 4$$
$$5x - 3x = 4 + 6$$
$$2x = 10$$
$$x = \frac{10}{2}$$
$$x = 5$$

4. Diagonals $AC = 24$ cm and $BD = 18$ cm



Since, diagonals of a rhombus bisect each other at right angles.

$$AO = \frac{1}{2} \times AC = \frac{1}{2} \times 24 = 12 \text{ cm}$$
$$BO = \frac{1}{2} \times BD = \frac{1}{2} \times 18 = 9 \text{ cm}$$

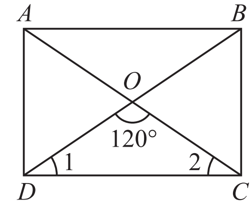
$\therefore$ In right angled ΔAOB

$$AB^2 = AO^2 + BO^2$$
$$= (12)^2 + (9)^2 = 144 + 81$$
$$AB^2 = 225$$
$$AB = \sqrt{225} = 15 \text{ cm}$$

So, the length of each side of the rhombus = 15 cm

Now, perimeter of rhombus = 4 × side = 4 × 15 = 60 cm

5. In rectangle $ABCD, \angle DOC = 120^\circ$ Since, diagonals of rectangle bisect each other



$\Rightarrow \quad OD = OC$

$\Rightarrow \quad \angle 1 = \angle 2$

$\therefore$ In ΔDOC,

$$120^\circ + \angle 1 + \angle 2 = 180^\circ$$
$$\angle 1 + \angle 1 = 180^\circ - 120^\circ$$
$$2\angle 1 = 60^\circ$$
$$\angle 1 = 30^\circ$$

$\therefore \quad \angle ODC = \angle OCD = 30^\circ$

6. Sum of the interior angles of pentagon = 540°

$$x° + (x-4)° + (2x-10)° + (2x-5)° + (2x+15)° = 540°$$

$$8x + (-19+15) = 540°$$

$$8x - 4 = 540°$$

$$8x = 540° + 4°$$

$$8x = 544°$$

$$x = \frac{544}{8}$$

$$x = 68°$$

Hence, $x° = 68°, (x-4) = (68-4) = 64°,$

$(2x-10)° = (2 \times 68 - 10)° = 126°,$

$(2x-5)° = (2 \times 68 - 5) = 131°$

and $(2x+15)° = (2 \times 68 + 15) = 151°$

7. Since, opposite angles of a ||gm are equal.

$$(3x+12)° = (2x+52)°$$

$$3x - 2x = 52° - 12°$$

$$x = 40°$$

So, all angles of ||gm are $40°, 40°, (180-40°)$ and $(180-40°)$

$\Rightarrow$ $40°, 40°, 140°$ and $140°$

8. (a) Since, sum of adjacent angles are 180°

$$10x + 8x = 180°$$

$$18x = 180°$$

$$x = \frac{180°}{18} = 10°$$

$\therefore$ $x = 10°$

Angle $y = 8x = 8 \times 10° = 80°$

Angle $z = 10x = 10 \times 10 = 100°$

(b) Angle $y = 40°$ (Alternate interior angles)

Angle $x = 180° - 80° = 100°$

$\therefore$ Angle $z = 180° - (100 + y)$ (Triangle sum property)

$= 180° - (100 + 40°) = 40°$

(c) Angle $y = 110°$ (Opposite angles ||^{gm})

$\therefore$ Angle $z = 180° - (30° + y)$ (Triangle sum property)

$= 180° - (30° + 110°)$

$= 180° - 140° = 40°$

$\therefore$ Angle $x = z = 40°$ (Alternate interior angles)

(d) Angle $x = 70°$ (Alternate interior angles)

Angle $y = 180° - (110° + 40°)$ (Triangle sum property)

$= 180° - 150° = 30°$

$\therefore$ Angle $z = y = 30°$ (Alternate interior angles)

(e) $\therefore$ $y + 100° = 180°$ (Sum of adjacent angles)

$y = 180° - 100°$

$y = 80°$

$\therefore$ $z + 70° = 180°$ (Sum of adjacent angles)

$z = 180° - 70°$

$z = 110°$

Now, $70° + 80° + x = 180°$ (Angle sum property)

$150° + x = 180°$

$x = 180° - 150°$

$x = 30°$

9. (a) Since, diagonals of $||^{gm}$ bisect each other,

$y + 3 = 10$ cm and $x + 7 = 18$ cm

$y = (10 - 3)$ cm $\quad x = (18 - 7)$ cm

$y = 7$ cm $\quad x = 11$ cm

(b) Since, opposite sides of a $||^{gm}$ are equal

$5x = 15$ and $2y - 6 = 18$

$x = \frac{15}{5}$ $\quad 2y = 18 + 6$

$x = 3$ $\quad 2y = 24$

$y = \frac{24}{2}$

$y = 12$

12. Practical Geometry

Do yourself.

13. Mensuration

Exercise 13.1

1. (a) Area of the rectangle $= l \times b$

$= 17 \times 9 \text{ cm}^2 = 153 \text{ cm}^2$

Perimeter of the rectangle $= 2 \times (l + b)$

$= 2 \times (17 + 9)$

$= 2 \times 26 = 52$ cm

(b) Area of semicircle $= \frac{\pi r^2}{2}$

$$= \frac{22}{7} \times \frac{7}{2} \times 7 = \frac{154}{2} \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

Perimeter of semicircle $= \pi r + d$

$$= \frac{22}{7} \times 7 + 2 \times r$$

$$= 22 + 2 \times 7$$

$$= 22 + 14 = 36 \text{ cm}$$

(c) Area of triangle $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$$

Perimeter of triangle = Sum of three sides of the triangle

$$= (10 + 12 + 15) \text{ cm} = 37 \text{ cm}$$

(d) Area of square = Side × Side

$$= 4 \text{ cm} \times 4 \text{ cm}$$

$$= 16 \text{ cm}^2$$

Perimeter of square = 4 × side

$$= 4 \times 4 \text{ cm} = 16 \text{ cm}$$

(e) Area of parallelogram = Base × Height

$$= 7 \times 5 \text{ cm}^2 = 35 \text{ cm}^2$$

Perimeter of parallelogram $= 2 \times (l + b)$

$$= 2 \times (7 + 4) \text{ cm}$$

$$= 2 \times 11 \text{ cm} = 22 \text{ cm}$$

2. Side of the square = 44 cm

Given, perimeter of the circle = Perimeter of the square

$$2\pi r = 4 \times \text{side}$$

$$2 \times \frac{22}{7} \times r = 4 \times 44$$

$$r = \frac{4 \times 44 \times 7}{2 \times 22}$$

$$r = 28 \text{ cm}$$

Hence, the radius of the circle is 28 cm.

3. Base of the triangle = 200 m

Area of the triangle $= 17500 \text{ m}^2$

Area of the triangle $= \frac{1}{2} \times b \times h$

$$17500 = \frac{1}{2} \times 200 \times h$$

$$h = \frac{2 \times 17500}{200}$$

$$h = 175 \text{ m}$$

Hence, the altitude of the field is 175 m.

4. The minute hand will go round the clock 48 times in 2 days.

The distance covered by the minute hand in 1 round = $2\pi r$

$$= 2 \times \frac{22}{7} \times 5.6$$

$$= 44 \times 0.8 = 35.2 \text{ cm}$$

$\therefore$ The distance covered by the minute hand in 48 rounds $= 35.2 \times 48$ cm

$$= 1689.6 \text{ cm}$$

5. Perimeter of rectangle $= 2 \times (l + b)$

$$= 2 \times [(250 - 2r) + 140]$$

$$= 2 \times \left[\left(250 - 2 \times \frac{140}{2}\right) + 140\right]$$

$$= 2 \times [(250 - 140) + 140] = 500 \text{ m}$$

Perimeter of semicircles $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{140}{2} = 440 \text{ m}$$

$\therefore$ Total perimeter of figure $= (500 + 440)$ m $= 940$ m

Total area of given figure = A of R + A of two semicircles

$$= l \times b + 2 \times \frac{\pi r^2}{2} \qquad \left[\because r = \frac{d}{2} = \frac{140}{2} = 70 \text{ m}\right]$$

$$= 110 \times 140 + \frac{22}{7} \times 70 \times 70$$

$$= 15400 + 15400 = 30800 \text{ m}^2$$

6. Length of the floor = 8.4 m

Breadth of the floor = 5.4 m

Area of the floor $= l \times b$

$$= 8.4 \times 5.4 \text{ m}^2$$

Now, base of the triangular piece = 40 cm = 0.4 cm

Height of the triangular piece = 30 cm = 0.3 cm

Area of the triangular piece $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 0.4 \times 0.3 \text{ m}^2$$

$$= 0.2 \times 0.3 \text{ m}^2$$

Required number of pieces $= \frac{\text{A of the floor}}{\text{A of the triangular piece}}$

$$= \frac{8.4 \times 5.4}{0.2 \times 0.3} = \frac{84 \times 54}{6} = 756$$

Hence, 756 marble pieces will be required to cover the floor.

7. First figure :

Perimeter of semicircle $= \pi r$

$$= \frac{22}{7} \times \frac{42}{2} = 22 \times 3 = 66 \text{ m}$$

$\therefore$ Cost $= ₹7 \times 66 = ₹462$

Second figure :

Perimeter of figure $= \pi r + P$ of triangle

$$= \frac{22}{7} \times \frac{28}{2} + (28 + 7 + 8)$$

$$= 22 \times 2 + 43$$

$$= 44 + 43 = 87 \text{ m}$$

$\therefore$ Cost $= ₹7 \times 87 = ₹609$

Third figure :

Perimeter of given figure $= P$ of semicircle $- P$ of rectangle

$$= \pi r + 2 \times (l + b)$$

$$= \frac{22}{7} \times \frac{14}{2} + 2 \times (14 + 9)$$

$$= 22 + 2 \times 23 = 22 + 46 = 68 \text{ m}$$

Cost $= ₹7 \times 68 = ₹476$

$\therefore$ Total cost $= ₹(462 + 609 + 476) = ₹1547$

Hence, second figure requires the longest fence.

Exercise 13.2

1. Area of trapezium $= 119 \text{ cm}^2$

Altitude $h = 7$ cm

Let the shorter parallel side be x cm

Then the longer parallel side $= (x + 8)$ cm

$\therefore$ Area of trapezium $= \frac{1}{2} \times h \times$ (sum of parallel sides)

$$119 = \frac{1}{2} \times 7 \times (x + x + 8)$$

$$119 = \frac{7}{2} \times (2x + 8)$$

$$119 = \frac{7}{2} \times 2(x + 4)$$

$$\frac{119}{7} = x + 4$$

$$17 = x + 4$$

$$x = 17 - 4$$

$$x = 13 \text{ cm}$$

$\therefore$ $x + 8 = 13 + 8 = 21$ cm

Hence, the parallel sides are 13 cm and 21 cm.

2. $\therefore$ Area of $\Delta QRT = \frac{1}{2} \times b \times h$

$$75 = \frac{1}{2} \times 15 \times h$$

$$h = \frac{75 \times 2}{15}$$

$$h = 10 \text{ cm}$$

Area of trapezium $PQRS = \frac{1}{2} \times h \times (PQ + RS)$

$$= \frac{1}{2} \times 10 \times (12 + 27)$$

$$= 5 \times 39 = 195 \text{ cm}^2$$

Hence, required area of trapezium $PQRS$ is 195 cm^2.

3. Longer side of trapezium $(a) = 80$ dm

Shorter side of trapezium $(b) = 6$ m

$$= 6 \times 10 \text{ dm} = 60 \text{ dm}$$

Altitude $(h) = 40$ dm

$\therefore$ Area of trapezium $= \frac{1}{2} \times h \times (a + b)$

$$= \frac{1}{2} \times 40 \times (80 + 60) \text{ dm}^2$$

$$= 20 \times 140 \text{ dm}^2 = 2800 \text{ dm}^2$$

Hence, the required area is 2800 dm^2.

4. First diagonal of rhombus $d_1 = 16$ cm

Second diagonal of rhombus $d_2 = 20$ cm

$\therefore$ Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} \times 16 \times 20 \text{ cm}^2$$

$$= 8 \times 20 \text{ cm}^2 = 160 \text{ cm}^2$$

Hence, the area of rhombus is 160 cm^2.

5. Diagonal $AC = 48$ cm

$DP\ (h_1) = 17.5$ cm

$BQ\ (h_2) = 12$ cm

$\therefore$ Area of quadrilateral $ABCD = \frac{1}{2} \times d \times (h_1 + h_2)$

$$= \frac{1}{2} \times 48 \times (17.5 + 12) \text{ cm}^2$$

$$= \frac{1}{2} \times 48 \times 29.5 \text{ cm}^2$$

$$= 24 \times 29.5 \text{ cm}^2 = 708 \text{ cm}^2$$

Hence, the required area of quadrilateral is 708 cm^2.

6. Area of the floor $= 43.2\ m^2$

Base of parallelogram (tile) $= 18$ cm

Height of parallelogram (tile) $= 8$ cm

Area of one tile $= b \times h$

$$= 18 \times 8\ cm^2 = 144\ cm^2$$

$\therefore$ No. of tiles required $= \dfrac{\text{Area of the floor}}{\text{Area of the tile}}$

$$= \frac{43.2 \times 100 \times 100\ cm^2}{144\ cm^2} = 3000 \text{ tiles}$$

Cost of covering the floor if the cost of each tile is ₹ 25 = ₹ 25 × 3000 = ₹ 75000

Hence, the total cost of covering the floor is ₹ 75000.

7. Area of the regular octagon $= 2 \times$ Area of trapezium + Area of rectangle

$$= 2 \times \frac{1}{2} \times h \times (a + b) + l \times b$$

$$= 3 \times (5 + 13) + 13 \times 5$$

$$= 3 \times 18 + 65$$

$$= 54 + 65 = 119\ cm^2$$

Hence, the area of th octagon is 119 cm^2.

8. Height of the triangle $= \dfrac{1}{2}(25 - 15)$

$$= \frac{1}{2} \times 10 = 5 \text{ cm}$$

$\therefore$ Area of the tile $= 2 \times$ Area of the triangle + Area of the rectangle

$$= 2 \times \left(\frac{1}{2} \times b \times h\right) + l \times b$$

$$= 2 \times \frac{1}{2} \times 28 \times 5 + 28 \times 15$$

$$= 140 + 420 = 560\ cm^2$$

Hence, the required area of the tile is 560 cm^2.

Exercise 13.3

1. Surface area of the cube $= 726\ cm^2$

Let the edge of the cube be a cm.

$\therefore$ Surface area of the cube $= 6\,a^2$

$$726 = 6a^2$$

$$a^2 = \frac{726}{6}$$

$$a = \sqrt{121}$$

$$a = 11 \text{ cm}$$

Hence, the length of the edge of the cube is 11 cm.

2. Length of the well = 2 m

Breadth of the well = 1.5 m

Height of the well = 10 m

Lateral surface area of the well = $2 \times h(l + b)$

$$= 2 \times 10 \times (2 + 1.5)$$

$$= 20 \times 3.5 = 70 \text{ m}^2$$

$\therefore$ Cost of cementing the walls of well at the rate of ₹ 52 per m^2

$$= ₹\, 52 \times 70 = ₹\, 3640$$

Hence, the required cost is ₹ 3640.

3. First figure is cylinder and the other figure is cube.

Lateral surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{9}{2} \times 9 = 254.57 \text{ cm}^2$$

Now, lateral surface area of cube = $4a^2$

$$= 4 \times 9 \times 9 = 324 \text{ cm}^2$$

No, the lateral surface area of both the figures are not same.

4. Since the outer surface of the box is to be painted, we need to find out its total surface area.

Length of the wooden box = 75 cm

Breadth of the wooden box = 60 cm

Height of the wooden box = 40 cm

Total surface area of the box = $2(lb + bh + hl)$

$$= 2 \times [(75 \times 60) + (60 \times 40) + (40 \times 75)]$$

$$= 2[4500 + 2400 + 3000]$$

$$= 2 \times 9900 = 19800 \text{ cm}^2$$

$\therefore$ Cost of painting the box = ₹ $\frac{19800 \times 15}{100}$ = ₹ 2970

Hence, the total cost of painting the box is ₹ 2970.

5. Given = $\frac{\text{Curved surface area of cylinder}}{\text{Total surface area of cylinder}} = \frac{5}{7}$

$$\frac{2\pi rh}{2\pi r(h + r)} = \frac{5}{7}$$

$$\frac{h}{h + r} = \frac{5}{7}$$

$$7h = 5h + 5r$$

$$7h - 5h = 5r$$

$$2h = 5r$$

$$\frac{h}{r} = \frac{5}{2}$$

$$h : r = 5 : 2$$

Hence, the required ratio is 5 : 2.

6. Total surface area of cuboid $= 160 \text{ m}^2$

Lateral surface area of cuboid $= 120 \text{ m}^2$

$$\therefore \quad 2(lb + bh + hl) = 160 \quad \ldots(1)$$

$$2h(l + b) = 120$$

$$2(bh + hl) = 120 \quad \ldots(2)$$

On subtracting equations (1) and (2)

$$2(lb + bh + hl) - 2(bh + hl) = 160 - 120$$

$$2lb + 2bh + 2hl - 2bh - 2hl = 40$$

$$2lb = 40$$

$$lb = \frac{40}{2}$$

$$lb = 20$$

Hence, area of base $= l \times b = 20 \text{ m}^2$

7. Length of classroom $= 7$ m

Breadth of classroom $= 6$ m

Height of classroom $= 4$ m

Surface area of classroom of whitewashing the walls and the roof

$$= 2(lb + bh - hl) - lb$$

$$= 2(7 \times 6 + 6 \times 4 - 4 \times 7) - 7 \times 6$$

$$= 2(42 + 24 - 28) - 42$$

$$= 2 \times 94 - 42$$

$$= 188 - 42 = 146 \text{ m}^2$$

Required surface area without doors and windows $= 146 - 7 = 139 \text{ m}^2$

$\therefore$ Cost of whitewashing at the rate ₹ 15 per m^2 = ₹ 139×15 = ₹ 2085

8. Length of the edge of cubical box = 20 cm

Surface area of 250 boxes $= 250 \times 6a^2$

$$= 250 \times 6 \times 20 \times 20 \text{ cm}^2$$

Area of cardboard sheets $= l \times b = 50 \times 50 \text{ cm}^2$

$\therefore$ Required number of cardboard sheets $= \dfrac{\text{Surface area of cubical boxes}}{\text{Area of cardboard sheets}}$

$$= \frac{250 \times 6 \times 20 \times 20}{50 \times 50} = 240 \text{ sheets}$$

Hence, 240 sheets are required to make the boxes.

9. Height of the cylindrical tank $= 8$ m

Radius of the cylindrical tank $= 3.5$ m

Surface area of closed cylindrical tank $= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 3.5 \times (8 + 3.5)$$

$$= 2 \times \frac{22}{7} \times \frac{3.5}{10} \times 11.5$$

$$= 22 \times 11.5 = 253.0 \text{ m}^2$$

$\therefore$ Cost of the metal sheet required at the rate of ₹ 130 per m^2 = ₹ 130×253

= ₹32.890

Hence, the required cost of metal sheet is ₹ 32980.

10. Length of the room = 24 m

Width of the room = 16 m

Area of the floor = $l \times b = 24 \times 16$

Area of the ceiling of the room = $l \times b = 24 \times 16$

$\therefore$ Area of 4 walls = $2h(l + b)$

$$= 2h(24 + 16)$$

Given : $24 \times 16 + 24 \times 16 = 2h(24 + 16)$

$$2 \times 24 \times 16 = 2 \times h \times 40$$

$$h = \frac{2 \times 24 \times 16}{2 \times 40}$$

$$= \frac{96}{10} = 9.6 \text{ m}$$

Hence, the height of the room is 9.6 m.

Exercise 13.4

1. Length of the paper = 66 cm

Breadth of the paper = 14 cm (Height of cylinder)

$\therefore$ Circumference of cylinder = length of the paper

$$2\pi r = 66$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \frac{66 \times 7}{2 \times 22}$$

$$r = \frac{21}{2} \text{ cm}$$

$\therefore$ Total surface area of cylinder = $2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times \frac{21}{2}\left(14 + \frac{21}{2}\right)$$

$$= 22 \times 3 \times \frac{49}{2}$$

$$= 33 \times 49 = 1617 \text{ cm}^2$$

$\therefore$ Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 14$$

$$= 11 \times 21 \times 21 = 4851 \text{ cm}^3$$

2. Area of the floor of the room $= l \times b = 85.5 \text{ m}^2$

Volume of the room $= l \times b \times h = 983.25 \text{ m}^3$

$$\therefore \quad \frac{l \times b \times h}{l \times b} = \frac{983.25}{85.5}$$

$$h = 11.5 \text{ m}$$

Hence, the height of the room is 11.5 m.

3. Height of the first cylinder $= 5x$

Height of the second cylinder $= 6x$

Radius of the first cylinder $= 4y$

Radius of the second cylinder $= 5y$

$\therefore$ Ratio of the volumes of the cylinders $= \dfrac{\text{Volume of the first cylinder}}{\text{Volume of the second cylinder}}$

$$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \frac{(4y)^2 \times 5x}{(5y)^2 \times 6x}$$

$$= \frac{4 \times 4 \times y^2 \times 5 \times x}{5 \times 5 \times y^2 \times 6 \times x}$$

$$= \frac{4 \times 4}{5 \times 6} = \frac{8}{15}$$

Hence, the required ratio is 8 : 15.

4. Curved surface area of cylinder $= 2640 \text{ cm}^2$

Circumference of the base $(2\pi r) = 66$ cm

$\therefore$ Curved surface area $= 2\pi rh$

$$2640 = 66 \times h$$

$$h = \frac{2640}{66}$$

$$h = 40 \text{ cm}$$

$$\therefore \quad 2\pi r = 66$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$$

$\therefore$ Volume of the right circular cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40$$

$$= 11 \times 3 \times 21 \times 20 = 13860 \text{ cm}^3$$

5. Side of cubical blocks $= 30$ cm

Number of cubical blocks $= \dfrac{\text{Volume of rectanglular piece}}{\text{Volume of cubical box}}$

$$= \frac{3 \text{ m} \times 75 \text{ cm} \times 60 \text{ cm}}{30 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm}}$$

$$= \frac{300 \text{ cm} \times 75 \text{ cm} \times 60 \text{ cm}}{30 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm}}$$

$$= \frac{3 \times 75 \times 6}{3 \times 3 \times 3} = 50 \text{ blocks}$$

Hence, the required number of metal cubical blocks are 50.

6. Total surface area of the cube $= 3750 \text{ cm}^2$

$\therefore$ Total surface area of the cube $= 6a^2$

$$3750 = 6a^2$$

$$a^2 = \frac{3750}{6}$$

$$a^2 = 625$$

$$a = \sqrt{625}$$

$$a = 25 \text{ cm}$$

$\therefore$ Volume of the cube $= a \times a \times a \text{ cm}^2$

$$= 25 \times 25 \times 25 \text{ cm}^2$$

$$= 15625 \text{ cm}^3$$

Hence, volume of the cube is 15625 cm^3.

7. Dimensions of hall $= 150 \text{ m} \times 85 \text{ m} \times 12 \text{ m}$

Volume of air $= 50 \text{ m}^3$

Required number of persons $= \dfrac{\text{Volume of hall}}{\text{Volume of air}}$

$$= \frac{150 \text{ m} \times 85 \text{ m} \times 12 \text{ m}}{50 \text{ m}^3}$$

$$= \frac{150 \times 85 \times 12}{50} = 3060$$

Hence, 3060 persons can sit in the hall.

8. Volume of earth dug out = Volume of the well

Depth of the well = 10 m

Radius of the well $= \dfrac{14}{2} = 7$ m

The well is cylindrical in shape, so

$$\text{Volume of well} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 10 = 1540 \text{ m}^3$$

The platform formed is cuboid in shape,

length = 100 m, breadth = 7 m

$\therefore$ Volume of the platform = Volume of the earth dug out

$$l \times b \times h = 1540$$

$$100 \times 7 \times h = 1540$$

$$h = \frac{1540}{100 \times 7}$$

$$h = 2.2 \text{ m}$$

Hence, the height of the platform formed is 2.2 m.

9. Length of water tank = 1.4 m

Width of water tank = 1 m

Depth of water tank = 0.7 m

Volume of tank = $l \times b \times h$

$= 1.4 \text{ m} \times 1 \text{ m} \times 0.7 \text{ m} = 0.98 \text{ m}^3$

$= 0.98 \times 1000$ litres = 980 litres

Hence, the water tank can hold 980 litres of water.

10. Length = 7.5 km = 7.5×1000 m = 750 m

Breadth = 100 m Height = 10 m

Volume of water = $l \times b \times h$

$$= \frac{750}{60 \times 60} \times 100 \times 10$$

$$= \frac{75 \times 100}{36} \text{ m}^3$$

$$= 208.33 \text{ litre}$$

14. Data Handling

Exercise 14.1

1. (a) School B (b) School A (c) School B

(d) School A = $\frac{60}{300} \times 100\% = 20\%$ (Total students in school A is 300)

School B = $\frac{70}{300} \times 100\%$ (Total students in school B is 300)

$= 23.33\%$

2. (a) Anis

(b) Average = $\frac{60 + 40 + 50 + 20 + 40}{5} = \frac{210}{5} = 42$

(c) Tallest student (Anis) = 60 cm

Shortest student (Deep) = 20 cm

$$\text{Ratio} = \frac{60}{20} = \frac{3}{1} = 3:1$$

3. (a) Number of babies weigh 3 kg or above = $25 + 30 + 5 + 2 = 62$

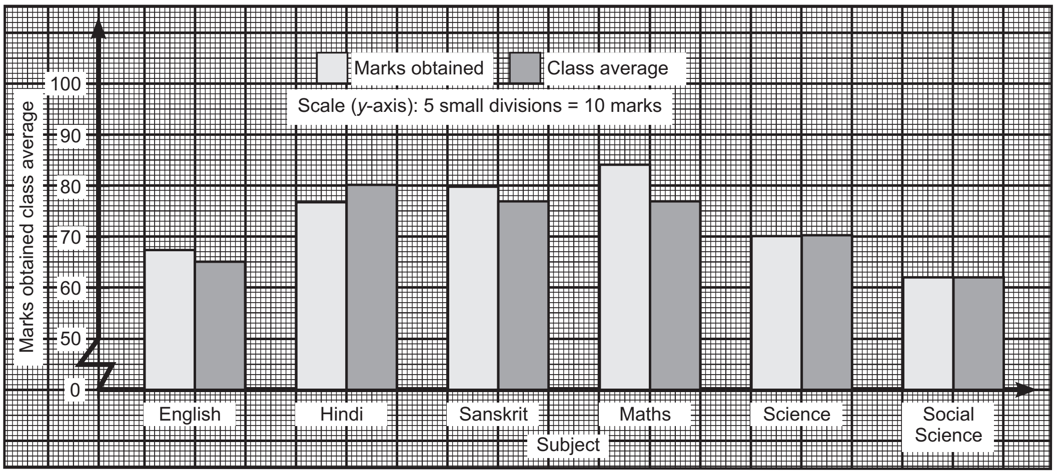
(b) Number of babies underweight 3 kg = $10 + 3 = 13$

(c) Number of babies weighing 5 kg or more but less than 6 kg = 5

Number of babies weighing 3 kg or more but less than 4 kg = 25

$\therefore$ $$\text{Ratio} = \frac{5}{25} = \frac{1}{5} = 1:5$$

4.



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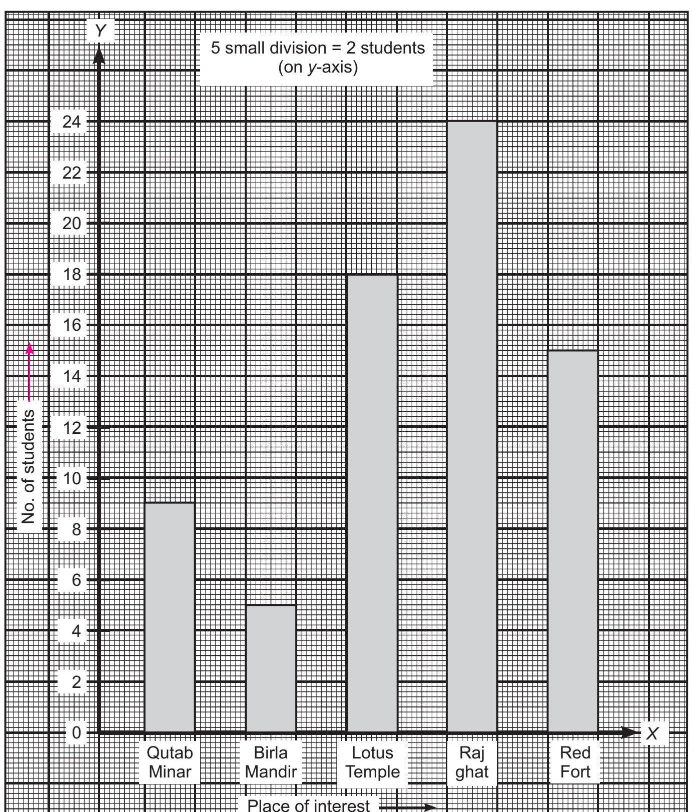

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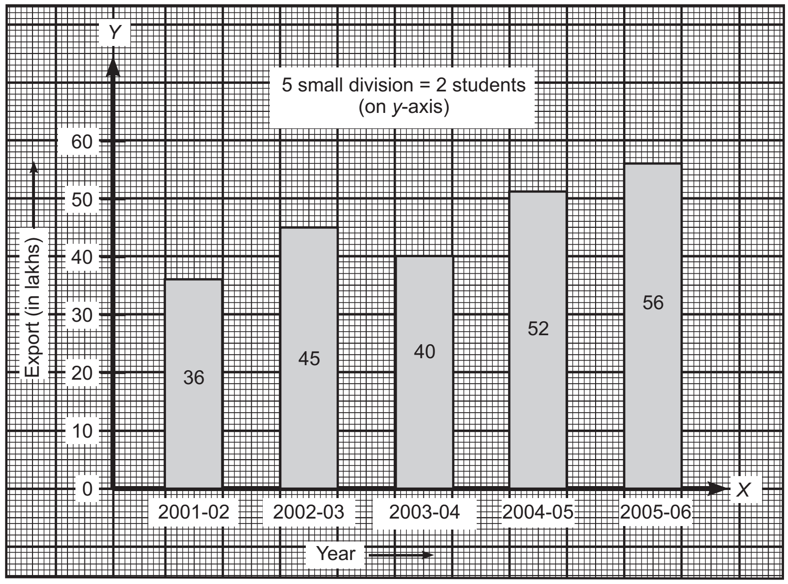


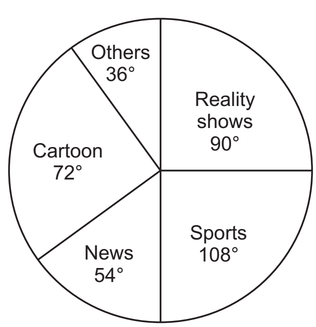
(a) Year 2005-06

(b) $\text{Average} = \dfrac{36 + 45 + 40 + 52 + 56}{5}$

$= \dfrac{229}{5} = 45.8 \text{ (in lakhs)}$

7.

TV programme	Number of persons	Measure of central angles
Sports	54	$\frac{54}{180} \times 360° = 108°$
Cartoons	36	$\frac{36}{180} \times 360° = 72°$
News	27	$\frac{27}{180} \times 360° = 54°$
Reality shows	45	$\frac{45}{180} \times 360° = 90°$
Others	18	$\frac{18}{180} \times 360° = 36°$
Total	180	360°



8. (a) For the total marks 540

Central angle $= 360°$

$\therefore$ For 120 marks, central angle $= \dfrac{360}{540} \times 120 = 80°$

Hence, central angle representing Hindi is $80°$, the students scored 120 marks in Hindi.

(b) The difference of central angle of Social Science and English $= 90° - 70° = 20°$

Corresponding difference of marks $= \dfrac{20}{360} \times 540° = 30$ marks

(c) Sum of central angles of Social Science and English $= 90° + 70° = 160°$

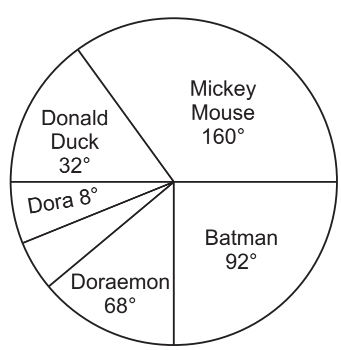
And sum of central angles of Science and Hindi $= 55° + 80° = 135°$

Hence, $135° < 160°$

So, the sum of marks in Science and Hindi is less than Social Science and English.

9.

Favourite cartoon characters	**Number of students**	**Measure of central angles**
Mickey Mouse	120	$\dfrac{120}{270°} \times 360° = 160°$
Batman	69	$\dfrac{69}{270°} \times 360° = 92°$
Doraemon	51	$\dfrac{51}{270°} \times 360° = 68°$
Dora	6	$\dfrac{6}{270°} \times 360° = 8°$
Donald Duck	24	$\dfrac{24}{270°} \times 360° = 32°$
Total	270	360°



Exercise 14.2

1. The outcomes of the given experiment

$$S = \{1, 2, 3, 4, 5, 6\}$$

$\therefore$ Total number of outcomes $= 6$

(a) Number less than 4 = {1, 2, 3}

$\therefore$ Number of favourable outcomes = 3

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(b) Prime numbers = {2, 3, 5}

$\therefore$ Number of favourable outcomes = 3

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(c) Number greater than 5 = {6}

$\therefore$ Number of favourable outcomes = 1

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6}$$

2. Number of red marbles = 2

Number of blue marbles = 3

Number of yellow marbles = 5

Number of green marbles = 1

Total number of marbles = 2 + 3 + 5 + 1 = 11

$\therefore$ Sample space S = {11}

Probability of getting a red marble,

$$P(E) = \frac{\text{Possible outcomes}}{\text{Total number of outcomes}} = \frac{2}{11}$$

Probability of getting a blue marble,

$$P(E) = \frac{3}{11}$$

Probability of getting a yellow marble,

$$P(E) = \frac{5}{11}$$

Probability of getting a green marble,

$$P(E) = \frac{1}{11}$$

3. The outcomes of the given experiment

$$S = \{1, 2, 3, 4, 5, 6\}$$

$\therefore$ Total number of outcomes = 6

(a) Even numbers = {2, 4, 6}

Number of favourable outcomes = 3

Number of getting an even number

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(b) Odd numbers = {1, 3, 5}

Number of favourable outcomes = 3

Probability of getting an odd number

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(c) Prime numbers = {2, 3, 5}

Number of favourable outcomes = 3

Probability of getting a prime number

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(d) Multiple of 3 = {3, 6}

Number of favourable outcomes = 2

Probability of getting a multiple of 3

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{2}{6} = \frac{1}{3}$$

4. Total number of outcomes = 52

(a) Ace of club = {1}

∴ Number of favourable outcomes = 1

Probability of getting an ace of club

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{52}$$

(b) Number of cards of heart = 13

∴ Number of favourable outcomes = 13

Probability of getting a heart

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{13}{52} = \frac{1}{4}$$

5. Number of red balls = 5

Number of blue balls = 5

Total number of balls = 10

∴ Total number of outcomes = 10

Probability of getting a red ball

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{5}{10} = \frac{1}{2}$$

6. If two unbiased coins are tossed simultaneously, we obtain any one of the following as an outcome HH, HT, TH, TT

∴ Total number of outcomes = 4

(a) One head is obtained if the outcomes HT, TH occurs.

∴ Number of favourable outcomes = 2

∴ Required probability $P(E) = \frac{2}{4} = \frac{1}{2}$

(b) No head is obtained if the outcome TT occurs.

∴ Number of favourable outcomes = 1

∴ Required probability $P(E) = \frac{1}{4}$

15. Visualising Solid Shapes

Exercise 15.1

1. (a) A hexagonal prism

A hexagonal prism has a polygon of 6 sides as its base.

Number of vertices $= 2 \times$ Number of sides in the base

$= 2 \times 6 = 12$

Number of edges $= 3 \times$ Number of sides in the base

$= 3 \times 6 = 18$

Number of faces $= 2 +$ Number of sides in the base

$= 2 + 6 = 8$

(b) An octagonal pyramid

An octagonal pyramid has a polygon of 8 sides as its base.

Number of vertices $= 1 +$ Number of sides in base

$= 1 + 8 = 9$

Number of edges $= 2 \times$ Number of sides in base

$= 2 \times 8 = 16$

Number of faces $= 1 +$ Number of sides in base

$= 1 + 8 = 9$

2. Name the solids,

(a) That have 5 faces and 5 vertices

$\Rightarrow$ Square pyramid

(b) That have 2 pentagonal faces and 5 rectangular faces.

$\Rightarrow$ Pentagonal prism

3. Number of edges $= 24$

Number of vertices $= 16$

Number of faces $= ?$

By Euler's formula,

$$F + V - E = 2$$
$$F + 16 - 24 = 2$$
$$F - 8 = 2$$
$$F = 2 + 8$$
$$F = 10$$

So, the number of faces is 10.

4. Number of faces $= 12$

Number of edges $= 20$

Number of vertices $= 14$

By Euler's formula

$$F + V - E = 2$$
$$12 + 14 - 20 = 2$$
$$26 - 20 = 2$$
$$6 \neq 2$$

Hence, Euler's formula is not satisfied. So, there is no polyhedron with 12 faces, 20 edges and 14 vertices.

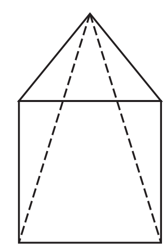
5. (a) 3, length, breadth, depth (b) Face
(c) 8 vertices, 6 faces (d) Vertex

6. Square pyramid is the solid whose number of faces and vertices are equal and the number of edges is three more than the number of faces.

Number of faces = 5

Number of vertices = 5

Number of edges = 8



Exercise 15.2

1. (a) (i) Side (ii) Front (iii) Top (b) (i) Front (ii) Side (iii) Top
(c) (i) Side (ii) Front (iii) Top (d) (i) Top (ii) Side (iii) Front

2. (a) Triangular prism (b) Cube
(c) Triangular prism (d) Square pyramid
(e) Hexagonal prism

16. Introduction to Graphs

Exercise 16.1

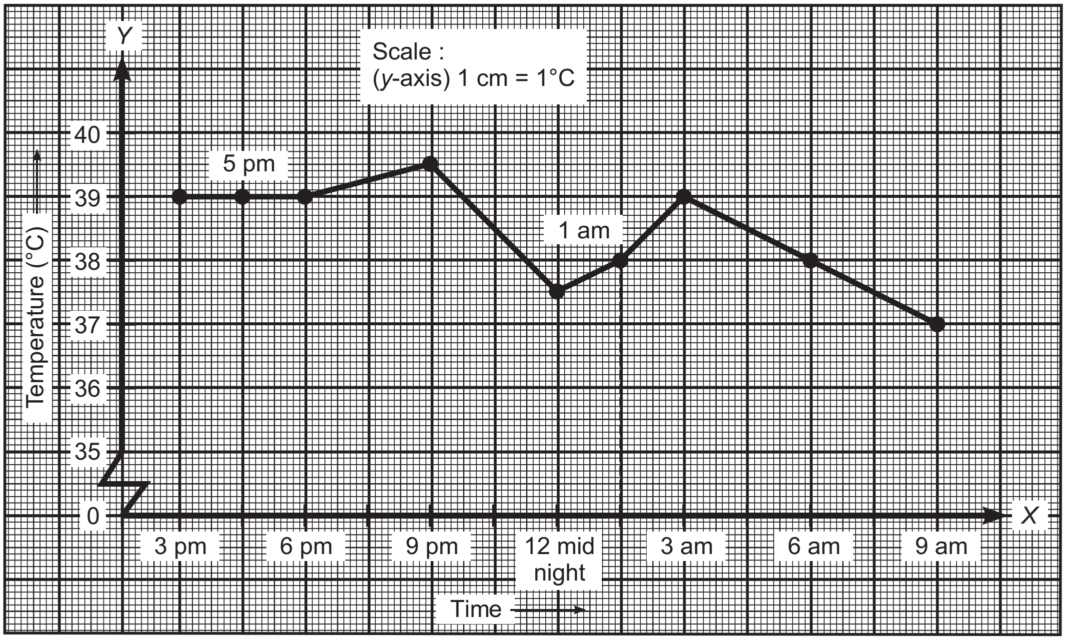
1. (a) 3 hrs
(b) Sunday and Friday
(c) Sunday → 4 hours, Saturday → 5 hours, ∴ difference = 5 – 4 = 1 hour
(d) Total no. of hours spent on the computer during the week

$$= 4 + 2.5 + 1 + 3 + 2 + 4 + 5 = 21.5 \text{ hours}$$

2. (a) May (b) January (c) Worse (d) April and June (e) January

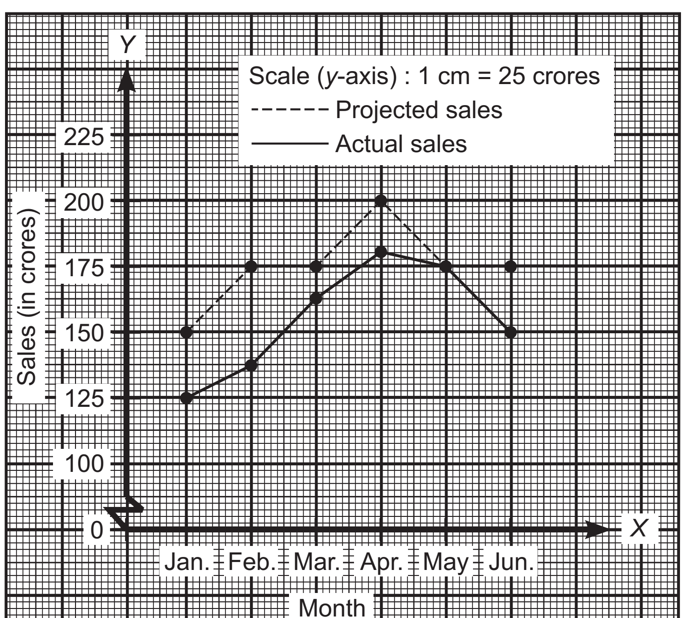
3. (a) The x-axis indicates the days of the week and the y-axis, the temperature recorded on all days of the two weeks.
(b) In the second week the temperature is on an average higher than the temperature in the first week.
(c) On Monday in the first week and on Thursday in the second week.
(d) On Thursday in the second week.
(e) On Saturday in both the weeks.

4. (a)

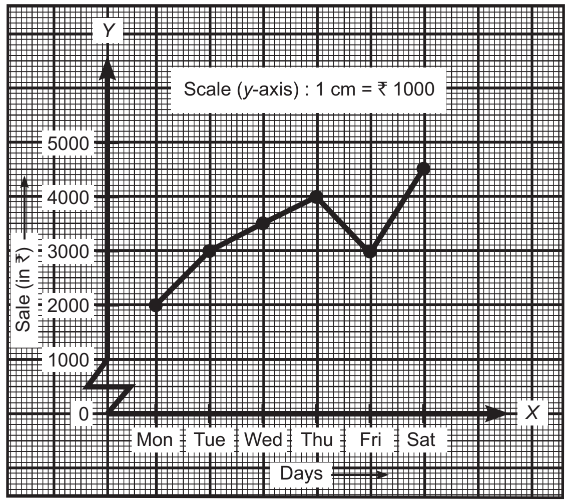


(b) 39°C (at 5 pm) and 38°C (at 1 am)

5.



6.



(a) Saturday

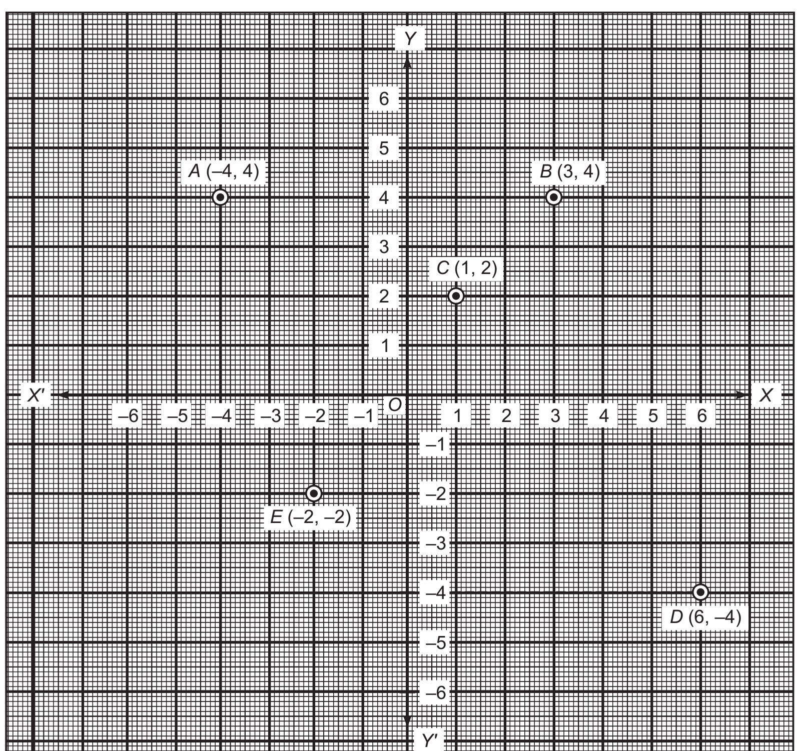
(b) Total collection during the week

$$= ₹(2000 + 3000 + 3500 + 4000 + 3000 + 4500) = ₹\,20000$$

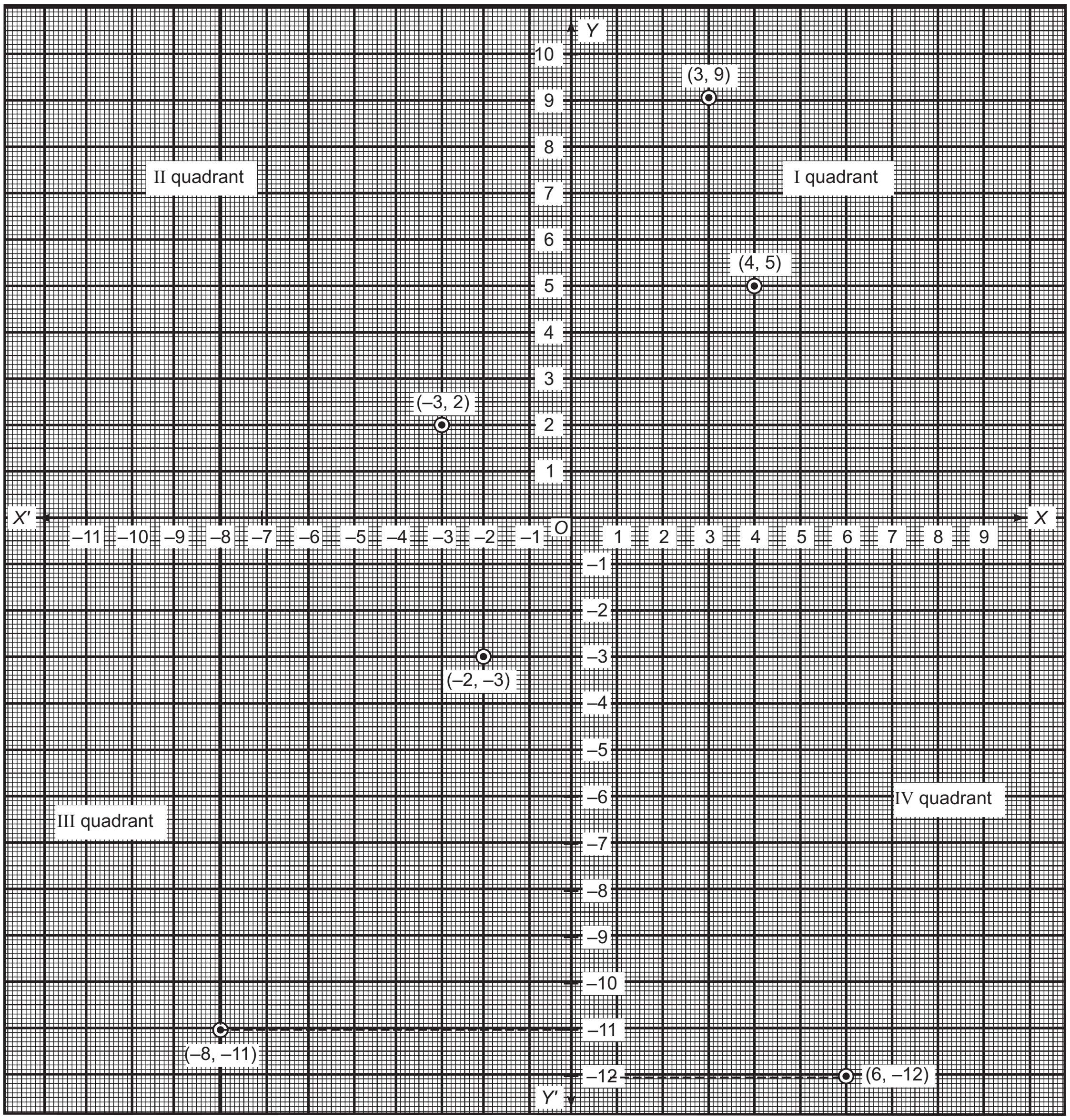
(c) Tuesday and Friday

Exercise 16.2

1.



2.



3. A(2, 2), B (7, 3), C (8, 6), D (4, 5)

4. (a) No, they don't lie on a straight line.

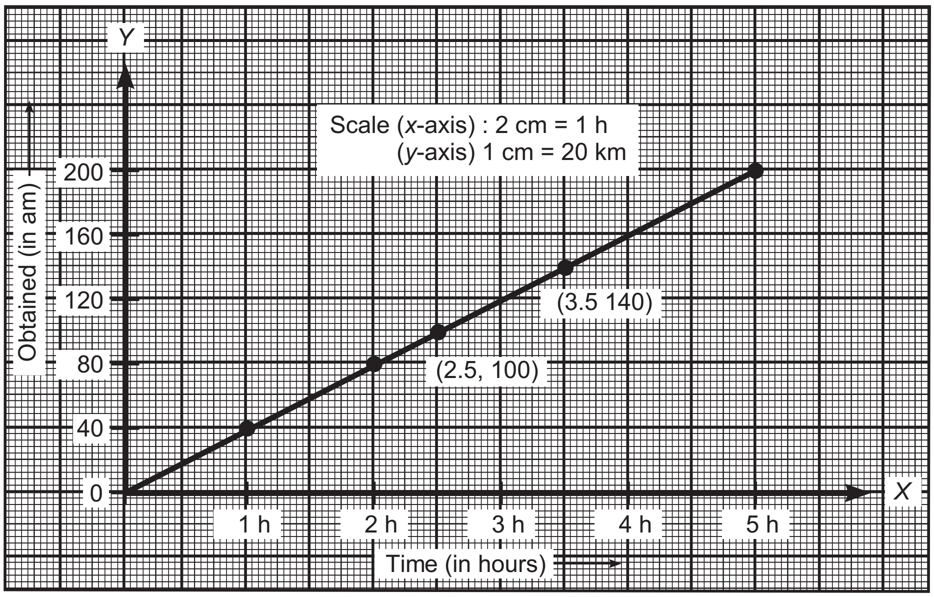
(b) Yes, they lie on a straight line

5. The relation between time (x hours) and distance travelled (y km) here is $y = 40\,x$. Taking time (x) = 1, 2, 3, 4, 5, we obtain values for distance (y) and make a table as follows.

Time (x) (h)	1	2	3	4	5
Distance (y) (km)	40	80	120	160	200

Taking scale of x-axis as 2 cm = 1 h and y-axis as 1 cm = 20 km, we plot the points (1, 40), (2, 80), (3, 120), (4, 160), (5, 200).

Joining the points we obtain a linear graph as shown in Figure.



(a) From the graph for $x = 3.5$, we get $y = 140$.

Therefore, distance travelled by Rohit in $3\frac{1}{2}$ hours is 140 km.

(b) And for $y = 100$, we get $x = 2.5$.

Time taken to travel 100 km is $2\frac{1}{2}$ hours.

6. The relationship between euro and rupees is as follows :

1 euro = ₹60

No. of euros	2	4	6	8	10
No. of rupees	120	240	360	480	600

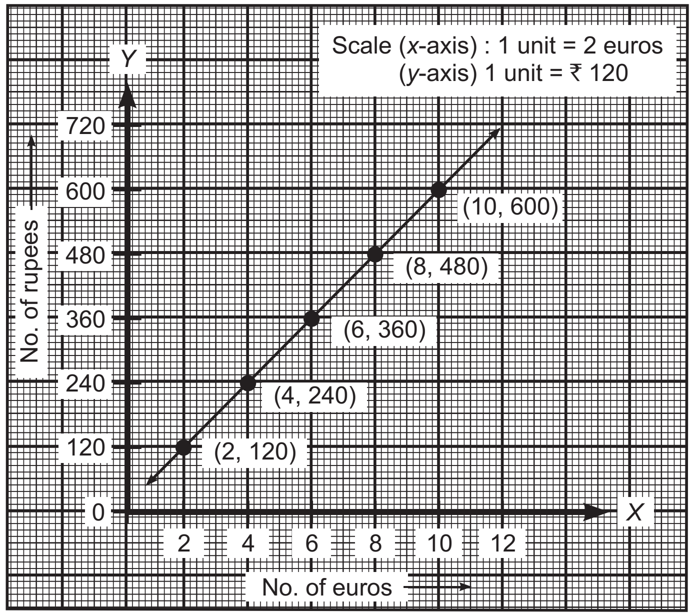
Let us plot a graph for the table shown above.

(i) Mark euros on the x-axis taking 1 unit as 1 euro.

(ii) Mark rupees on the y-axis taking 1 unit as 60 rupees.

(iii) Plot the points (1, 60), (2, 120), (3, 180), (4, 240), (5, 300).

(iv) Join these points and we get a linear graph showing direct variation between the variables.

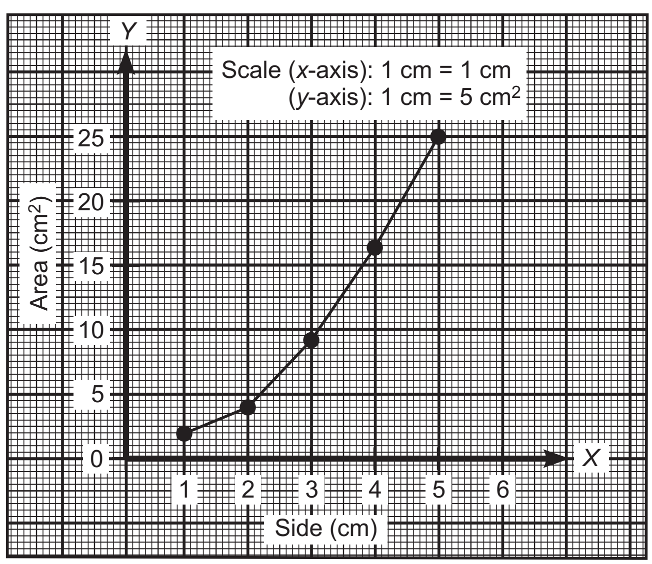


Let us now answer the question from the graph obtained.

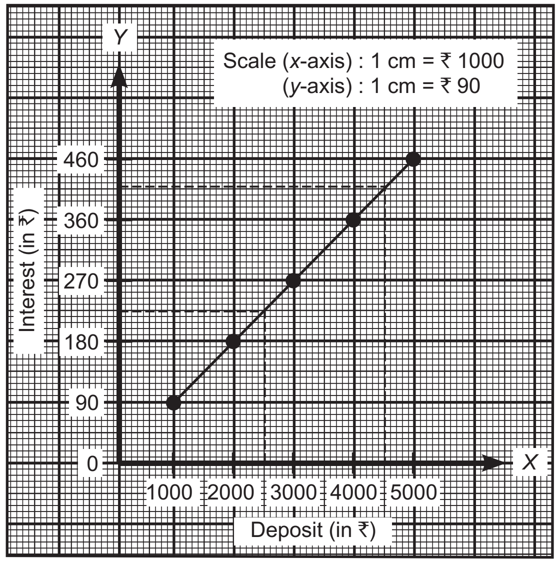
(a) We can make out that ₹ 720 = 12 euros by drawing a line from ₹ 720 until it reaches the graph line and then across to the x-axis.

(b) Draw a line vertically up from 11 euros to meet the graph line and then across the y-axis. It can be easily seen that 11 euros = ₹ 660.

7.



8.



(a) If principal is ₹ 4500, interest = ₹ 405

(b) If interest is 225, peincipal = ₹ 2500

❑